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Two Theorems on Generalized Meijer-Laplace Transform. (**)

1. – Generalizations of the LAPLACE transform

(1.1)
$$\varphi(s) = s \int_0^\infty e^{-st} f(t)\, dt,$$

have been given by various mathematicians, some of them are as follows:

(i) The MEIJER transform [11] is

(1.2)
$$\varphi(s) = s \int_0^\infty e^{-\frac{1}{2}st} (st)^{-k-\frac{1}{2}} W_{k+\frac{1}{2},m}(st) f(t)\, dt.$$

(ii) The generalization due to BOAS [2] is

(1.3)
$$\varphi(s) = \int_0^\infty g(s,\ t) f(t)\, dt.$$

(iii) The generalization due to VARMA [15] is

(1.4)
$$\varphi(s) = s \int_0^\infty e^{-\frac{1}{2}st} (st)^{m-\frac{1}{2}} W_{k,m}(st) f(t)\, dt.$$

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Recently MAINRA [8] gave another generalization in the form

(1.5) $$\varphi(s) = s\int_0^\infty e^{-\frac{1}{2}st}\,(st)^{-\lambda-\frac{1}{2}}\,W_{k+\frac{1}{2},m}(st)\,f(t)\,dt,$$

called the generalized MEIJER-LAPLACE transform and is denoted symbolically by

$$\varphi\left(s : k+\frac{1}{2},\ \lambda,\ m\right) = W\left[f(t);\ k+\frac{1}{2},\ \lambda,\ m\right],$$

where $\varphi\left(s : k+\frac{1}{2},\ \lambda,\ m\right) = \varphi(s)$, while (1.1) is denoted by $\varphi(s) \doteqdot f(t)$.

It is evident that (1.5) reduces to (1.2) if $\lambda = k$, and to (1.4) if $\lambda = -m$. Further, if $\lambda = k = -m$, (1.5) reduces to (1.1) due to the identity

$$z^{m-\frac{1}{2}}\,W_{\frac{1}{2}-m,m}(z) = e^{-\frac{1}{2}z}.$$

The object of this paper is to extend certain results of LAPLACE transform to this generalized MEIJER-LAPLACE transform. The results have been given in the form of theorems, with various particular cases.

2. – First we obtain generalized MEIJER-LAPLACE transform of some functions which will be needed in the subsequent investigations.

(a) Let $f(t) = t^{\nu}\,e^{-at}$.

Then $$\varphi(s) = s\int_0^\infty e^{-\frac{1}{2}st}\,(st)^{-\lambda-\frac{1}{2}}\,W_{k+\frac{1}{2},m}(st)\,t^{\nu}\,e^{-at}\,dt.$$

Now, using the integral due to GOLDSTEIN [6], namely

(2.1) $$\int_0^\infty x^{l-1}\,e^{-(\frac{1}{2}+\alpha^2)x}\,W_{k,m}(x)\,dx = \frac{\Gamma_{\times}(l+\frac{1}{2}\pm m)}{\Gamma(l-k+1)}\,{}_2F_1\left\{\begin{matrix} l+\frac{1}{2}+m,\ l+\frac{1}{2}-m \\ l-k+1 \end{matrix};-\alpha^2\right\}$$ (1),

where $R\left(l+\frac{1}{2}\pm m\right)>0$, $R(\alpha^2)+1>0$ and $|\alpha|<1$, and putting $\alpha^2 = a/s$ in (2.1), we find that

(2.2) $$\varphi(s) = \frac{\Gamma_{\times}(\nu+1-\lambda\pm m)}{\Gamma(\nu+1-\lambda-k)}\,s^{-\nu}\,{}_2F_1\left\{\begin{matrix} \nu+1-\lambda\pm m \\ \nu+1-\lambda-k \end{matrix};-a/s\right\},$$

provided $R(\nu+1-\lambda\pm m)>0$, $R(s+a)>0$ and $|s|>|a|$.

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(1) $\Gamma_{\times}(a\pm b) = \Gamma(a+b)\cdot\Gamma(a-b)$.

(b) Let $f(t) = t^{\mu} J_{\nu}(2\beta\sqrt{t})$.

Then $\varphi(s) = s \int_0^\infty e^{-\frac{1}{2}st} (st)^{-\lambda-\frac{1}{2}} W_{k+\frac{1}{2},m}(st)\, t^{\mu} J_{\nu}(2\beta\sqrt{t})\, dt$.

Expanding $J_{\nu}(2\beta\sqrt{t})$ in ascending powers of t and changing the order of integration and summation, provided it is justified, with the help of (2.1), we get

$$\varphi(s) = \frac{\beta^{\nu}\, \Gamma_x(\mu + 1 + (\nu/2) - \lambda \pm m)}{s^{\mu+(\nu/2)}\, \Gamma(\nu+1)\, \Gamma(\mu + (\nu/2) + 1 - \lambda - k)} \cdot \cdot F_2\left\{\begin{matrix}\mu + (\nu/2) + 1 - \lambda \pm m \\ \nu+1,\ \mu + (\nu/2) + 1 - \lambda - k\end{matrix}; -\beta^2/s\right\}, \tag{2.3}$$

provided that $\mathrm{R}(\mu + (\nu/2) + 1 - \lambda \pm m) > 0$ and $\mathrm{R}(s) > 0$.
The inversion in the order of integration is justified since,

(i) the infinite integral is absolutely and uniformly convergent when $\mathrm{R}(\mu + 1 + (\nu/2) - \lambda \pm m) > 0$ and $\mathrm{R}(s) > 0$.

(ii) the infinite series is uniformly and absolutely convergent and

(iii) the resulting series is absolutely convergent.

It is possible to obtain the original of some functions, by interpretation which can be expanded in descending powers of s.

(i) Let $\varphi(s) = \left(\frac{\alpha}{s}\right)^{\nu} e^{-\alpha/s} = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!} \left(\frac{\alpha}{s}\right)^{\nu+r}$.

Then interpreting the right hand side term by term by the relation

$$W\left[t^{\nu};\ k + \frac{1}{2},\ \lambda, m\right] = \frac{\Gamma_x(\nu + 1 - \lambda \pm m)}{\Gamma(\nu - \lambda - k + 1)}\, s^{-\nu}, \tag{2.4}$$

where $\mathrm{R}(\nu - \lambda + 1 \pm m) > 0$ and $\mathrm{R}(s) > 0$, we get

$$W\left[\frac{\Gamma(\nu - \lambda - k + 1)}{\Gamma_x(\nu + 1 - \lambda \pm m)} (\alpha t)^{\nu}\, {}_1F_2\left\{\begin{matrix}\nu-\lambda-k+1 \\ \nu-\lambda+1\pm m\end{matrix}; -\alpha t\right\};\ k + \frac{1}{2},\ \lambda, m\right] = \left(\frac{a}{s}\right)^{\nu} e^{-\alpha/s}. \tag{2.5}$$

(ii) $\varphi(s) = s^{\mu} J_{\nu}(a/\sqrt{s}) = \left(\frac{a}{2}\right)^{\nu} \sum_{r=0}^{\infty} \frac{(-1)^r (a/2)^{2r}}{r!\, \Gamma(1 + \nu + r)}\, s^{\mu-(\nu/2)-r}$.

Then interpreting term by term, with the help of (2.4) we get

$$W\left[\frac{a^{\nu}\,\Gamma((\nu/2)-\mu-\lambda-k+1)\,t^{(\nu/2)-\mu}}{2^{\nu}\,\Gamma_x((\nu/2)-\mu-\lambda+1\pm m)\,\Gamma(\nu+1)}\,{}_1F_3\left\{\begin{matrix}(\nu/2)-\mu-\lambda-k+1\\ (\nu/2)-\mu-\lambda+1\pm m,\nu+1\end{matrix};-a^2t^2/4\right\};\; k+\frac{1}{2},\ \lambda,\ m\right]$$

(2.6) $$= s^{\mu}\,J_{\nu}(a/\sqrt{s}),$$

provided that $\mathrm{R}((\nu/2)-\mu-\lambda+1\pm m)>0$ and $\mathrm{R}(s)>0$.

3. - Theorem 1. *If*

(3.1) $$W\left[t^{\nu} f(t);\ k+\frac{1}{2},\ \lambda,\ m\right]=\varphi(s),$$

and

(3.2) $$W\left[g(t);\ k'+\frac{1}{2},\ \lambda',\ m'\right]=f(s),$$

then

(3.3) $$\varphi(s)=\int_0^{\infty} g(u)\,\psi(u,\ s)\,\mathrm{d}x,$$

where

(3.4) $$\psi(u\ ,\ s)=\frac{1}{u^{\nu+1}}\,G_{33}^{22}\left(\frac{u}{s}\middle|\begin{matrix}\lambda+m,\ 1+\nu-\lambda'-k'\\ 1+\nu-\lambda'\pm m',\ \lambda+k\end{matrix}\right),$$

provided that $\mathrm{R}(\nu-\lambda-\lambda')+2>|\mathrm{R}(m)|+|\mathrm{R}(m')|$ *and* $g(t)$ *is a continuous function for* $t>0$, $\mathrm{R}(s)>0$, *the generalized Meijer-Laplace transform of* $|t^{\nu} f(t)|$ *and* $|g(t)|$ *exist and* (3.3) *converge absolutely.*

Proof. From (1.5), we have

(3.5) $$\varphi(s)=s\int_0^{\infty} e^{-\frac{1}{2}st}\,(st)^{-\lambda-\frac{1}{2}}\,W_{k+\frac{1}{2},m}(st)\,t^{\nu} f(t)\,\mathrm{d}t$$

and

(3.6) $$f(t)=t\int_0^{\infty} e^{-\frac{1}{2}ut}\,(ut)^{-\lambda'-\frac{1}{2}}\,W_{k'+\frac{1}{2},m'}(ut)\,g(u)\,\mathrm{d}x\,.$$

Substituting the value of $f(t)$ from (3.6) in (3.5) and changing the order of integration, provided it is justified, we get

$$(3.7)\qquad \varphi(s)=s^{-\lambda-\frac{1}{2}}\int_0^\infty u^{-\lambda'-\frac{1}{2}}\,g(u)\left[\int_0^\infty t^{\nu-\lambda-\lambda'}\,e^{-\frac{1}{2}(s+u)t}\,W^{k+\frac{1}{2},m}(st)\,W^{k'+\frac{1}{2},m'}(ut)\,dt\right]dx.$$

Evaluating the integral by a known formula [5], we get

$$\varphi(s)=\int_0^\infty g(u)\,\psi(u,\,s)\,dx,$$

where $\psi(u,\,s)=\frac{1}{u^{\nu+1}}\,G_{33}^{22}\left(\frac{u}{s}\,\middle|\,\begin{matrix}\lambda\pm m,\ 1+\nu-\lambda'-k'\\ 1+\nu-\lambda'\pm m',\ \lambda+k\end{matrix}\right)$. This proves the Theorem.

The inversion in the order of integration is easily justifiable as in (3.7) the t-integral converges absolutely and uniformly if $R(s)\geqslant s_0>0$ and $R(\nu-\lambda-\lambda')+2>|R(m)|+|R(m')|$. The u-integral converges absolutely and uniformly due to the existence of the generalised MEIJER-LAPLACE transform of $|g(t)|$ and the repeated integral converges absolutely due to the absolute convergence of (3.3). Thus the inversion in the order of integration is justified by DE LA VALLÉE POUSSIN's Theorem [3].

Corollaries:

(i) *If* $\nu=0$, $\lambda=k=-m$ *and* $\lambda'=k'=-m'$, *we get a result in Laplace transform* [9].

If $\varphi(s)\doteqdot f(t)$ *and* $f(s)\doteqdot g(t)$, *then* $\varphi(s)=s\int_0^\infty\frac{g(u)\,du}{(s+u)^2}$, *under suitable conditions.*

(ii) *If* $\lambda=k$, *we get a result due to Mehra* [10].

If $W\left[t^\nu f(t);\ k+\frac{1}{2},\ m\right]=\varphi(s)$, *and* $W\left[g(t);\ k'+\frac{1}{2},\ m\right]=f(s)$, *then*

$$\varphi(s)=\int_0^\infty g(u)\,\psi(u,\,s)\,dx,$$

where $\psi(u,\,s)=\frac{1}{u^{\nu+1}}\,G_{33}^{22}\left(\frac{u}{s}\,\middle|\,\begin{matrix}k\pm m,\ 1+\nu-2k'\\ 1+\nu-k'\pm m',\ 2k\end{matrix}\right)$.

(iii) *If* $\nu = 0$, $\lambda = k = -m$ *and* $\lambda' = k'$, *the theorem reduces to a result due to* *Jaiswal* [7].

If $\varphi(s) \doteqdot f(t)$ *and* $W\left[g(t);\ k + \frac{1}{2},\ m\right] = f(s)$, *then*

$$\varphi(s) = \int_0^\infty g(u)\ \psi(u,\ s)\ \mathrm{d}x,$$

where $\psi(u,\ s) = \dfrac{s\Gamma_x(\nu - k \pm m)}{u^2\ \Gamma(2 - 2k)}\ {}_2F_1\left\{\begin{matrix} 2 - k + m \\ 2 - 2k \end{matrix}\ ; - s/u\right\}.$

(iv) *If* $\nu = 0$, $\lambda' = k' = -m'$ *and* $\lambda = k$, *we get a result due io* *Bhonsle* [1].

If $W\left[f(t);\ k + \frac{1}{2},\ m\right] = \varphi(s)$ *and* $f(s) \doteqdot g(t)$, *then*

$$\varphi(s) = \int_0^\infty g(u)\ \psi(u,\ s)\ \mathrm{d}x,$$

where $\psi(u,\ s) = \dfrac{\Gamma_x(2 - k \pm m)}{s\Gamma(2 - 2k)}\ {}_2F_1\left\{\begin{matrix} 2 - k \pm m \\ 2 - 2k \end{matrix}\ ; - u/s\right\}.$

4. - Theorem 2 *If*

$$W\left[f(t);\ k + \frac{1}{2},\ \lambda,\ m\right] = \varphi(s) \tag{4.1}$$

and

$$W\left[g(t);\ k + \frac{1}{2},\ \lambda,\ m\right] = \sqrt{s}\ f(1/s), \tag{4.2}$$

then

$$\varphi\left(s^2 : k + \frac{1}{2},\ \lambda,\ m\right) = \frac{\sqrt{\pi}}{2^{1-2\lambda-2k}}\ W\left[tg\ (t^2/4);\ 2k + \frac{1}{2},\ 2\lambda,\ 2m\right], \tag{4.3}$$

provided $\mathrm{R}(s) > 0$. *The generalised Meijer-Laplace transform of* $|\,g(t)\,|$ *exists and* (4.3) *converges absolutely.*

Proof. From (1.5) we have

$$\varphi(s) = s \int_0^\infty e^{-\frac{1}{2}st}\ (st)^{-\lambda-\frac{1}{2}}\ W_{k+\frac{1}{2},m}(st)\ f(t)\ \mathrm{d}t$$

and

$$\sqrt{s}\ f(1/s) = s \int_0^\infty e^{-\frac{1}{2}sx} (sx)^{-\lambda-\frac{1}{2}}\ W_{k+\frac{1}{2},m}(sx)\ g(x)\ \mathrm{d}x.$$

On writing t for $1/s$, we get

$$f(t) = t^{-\frac{1}{2}} \int_0^\infty e^{-\frac{1}{2}x/t} (x/t)^{-\lambda-\frac{1}{2}}\ W_{k+\frac{1}{2},m}(x/t)\ g(x)\ \mathrm{d}x.$$

Therefore,

$$\varphi(s) = s \int_0^\infty e^{-\frac{1}{2}st} (st)^{-\lambda-\frac{1}{2}}\ W_{k+\frac{1}{2},m}(st)\ \mathrm{d}t \quad t^{-\frac{1}{2}} \int_0^\infty e^{-\frac{1}{2}x/t}\ W_{k+\frac{1}{2},m}(x/t)\ g(x)\ \mathrm{d}x.$$

Changing the order of integration, provided it is justified, we get

$$\varphi(s) = s \int_0^\infty (sx)^{-\lambda-\frac{1}{2}}\ g(x) \left[\int_0^\infty t^{-\frac{1}{2}}\ e^{-\frac{1}{2}(st+(x/t))}\ W_{k+\frac{1}{2},m}(x/t)\ \ W_{k+\frac{1}{2},m}(st)\ \ \mathrm{d}t \right] \mathrm{d}x.$$

Evaluating the integral by a known result [**5**], namely

$$\int_0^\infty x^{-\frac{1}{2}} \exp\left[-\tfrac{1}{2}\{(x/\alpha) + (\beta/x)\}\right] W_{k,\mu}(x/t)\ W_{k,\mu}(\beta/x)\ \mathrm{d}x$$

$$= \sqrt{\pi}\ 2^{\frac{1}{2}-2k} (\alpha\beta)^{1/4} \exp(-\beta/\alpha)^{\frac{1}{2}}\ W_{2k-\frac{1}{2},2m}(2\sqrt{\beta/\alpha}),$$

where $\mathrm{R}(\alpha) > 0$ and $\mathrm{R}(\beta) > 0$, we find that

$$\varphi(s) = s \int_0^\infty (st)^{-\lambda-\frac{1}{2}}\ g(t) \sqrt{\pi}\ 2^{\frac{1}{2}-2k} (t/s)^{1/4}\ e^{-\sqrt{st}}\ W_{2k+\frac{1}{2},2m}(2\sqrt{s}\ t)\ \mathrm{d}t,$$

provided $\mathrm{R}(s) > 0$ and $\mathrm{R}(x) > 0$. On writing s^2 for s and $t^2/4$ for t, we get

$$\varphi(s^2) = \frac{\sqrt{\pi}}{2^{2k-2\lambda+1}}\ W\left[t\ g(t^2/4);\ 2k + \frac{1}{2},\ 2\lambda,\ 2m\right].$$

The inversion in the order of integration is justified by DE LA VALLÉE POUSSIN's Theorem [3], under the conditions stated in the theorem.

Corollaries:

(i) *If* $\lambda = k = -m$, *we get a result in Laplace transform* [**9**].

If $\varphi(s) \doteqdot f(t)$ *and* $\Gamma(s)\ f(1/s) \doteqdot g(t)$, *then*

$$\varphi(s^2) \doteqdot (\sqrt{\pi}/2)\ t\ g(t^2/4).$$

(ii) *If* $\lambda = - m$ *and* $k + \frac{1}{2} = k'$, *we arrive at the following* [**12**].

If $\varphi(s : k', m) = W[f(t); k', m]$ *and* $\sqrt{s} f(1/s) = W[g(t); k', m]$, *then*

$$\varphi(s^2 : (k'/2) + (1/4), \; m/2) = W[\sqrt{\pi}\, 2^{-\frac{1}{2}-k-m}\, t\, g(t^2/4) \; ; \; k', \; m].$$

(iii) *If* $\lambda = k$, *we get the following result due to Mehra* [**10**].

If $W\left[f(t); \; k + \frac{1}{2}, \; m\right] = \varphi(s)$ *and* $W\left[g(t); \; k + \frac{1}{2}, \; m\right] = \sqrt{s}\, f(1/s)$, *then*

$$\varphi(s^2) = \int_0^\infty g(x)\, \psi(x, s)\, dx,$$

where $\psi(x, s) = \sqrt{\pi s/2}\left\{1/(2s)^{2k}\right\}[x^{-k-(1/4)}\, e^{-s\sqrt{x}}\, W_{2k+\frac{1}{2}, 2m}(2s\sqrt{x})]$.

For illustrating the theorem, the following example is worth mention.

Let $f(t) = t^\mu J_\nu(2\beta\sqrt{t})$. Then by (2.3), we have

$$\varphi(s) = \frac{\beta^\nu \Gamma(\mu + (\nu/2) + 1 - \lambda \pm m)}{s^{\mu+(\nu/2)}\, \Gamma(\nu+1)\, \Gamma(\mu+1+(\nu/2)-\lambda-k)}\, {}_2F_2\left\{\begin{matrix} \mu + (\nu/2) - \lambda + 1 \pm m \\ \nu+1, \; \mu+(\nu/2)+1-\lambda-k \end{matrix}; -\beta^2/s\right\},$$

where $R(\mu + (\nu/2) + 1 - \lambda \pm m) > 0$ and $R(s) > 0$. Also, $\sqrt{s}\, f(1/s) = s^{\frac{1}{2}-\mu} \cdot$ $\cdot\, J_\nu(2\beta/\sqrt{s})$, with the help of (2.6), we have

$$\sqrt{s}\, f(1/s) = W\left[\frac{\beta^\nu \Gamma(\mu + (\nu/2) + \frac{1}{2} - \lambda - k)}{\Gamma_x(\mu + (\nu/2) + \frac{1}{2} - \lambda \pm m)\, \Gamma(\nu + 1)}\, t^{\mu+(\nu/2)-\frac{1}{2}} \cdot \right.$$

$$\left. \cdot\, {}_1F_3\left\{\begin{matrix} \mu + (\nu/2) - \lambda - k + \frac{1}{2} \\ \nu + 1, \; \mu + (\nu/2) + \frac{1}{2} - \lambda \pm m \end{matrix}; -\beta^2 t\right\}; \; k + \frac{1}{2}, \; \lambda, \; m\right] = g(t),$$

provided $R\left(\mu + (\nu/2) + \frac{1}{2} - \lambda \pm m\right) > 0$ and $R(s) > 0$. Hence, by Theorem 2

$$W\left[t\, g(t^2/4); \; 2k + \frac{1}{2}, \; 2\lambda, \; 2m\right] = \left(\frac{2^{2k-2\lambda+1}}{\sqrt{\pi}}\right) \varphi\left[s^2 : k + \frac{1}{2}, \; \lambda, \; m\right].$$

The left hand side is

$$W\left[t\, g(t^2/4);\ 2k+\frac{1}{2},\ 2\lambda,\ 2m\right]$$

$$= \frac{\beta^{\nu}\, \Gamma\left(\mu + (\nu/2) + \frac{1}{2} - \lambda - k\right)}{\Gamma_{\times}\left(\mu + (\nu/2) + \frac{1}{2} - \lambda \pm m\right) \Gamma(\nu+1)}\, s \int_0^{\infty} e^{-\frac{1}{2}st} (st)^{-2\lambda - \frac{1}{2}}\, W_{2k+\frac{1}{2}, 2m}(st) \cdot$$

$$\cdot \frac{t^{2\mu+\nu}}{2^{2\mu+\nu-1}}\, {}_1F_3 \left\{ \begin{matrix} \mu + (\nu/2) - \lambda - k + \frac{1}{2} \\ \nu+1,\ \mu + (\nu/2) + \frac{1}{2} - \lambda \pm m \end{matrix} ;\ -\beta^2 t^2/4 \right\} dt$$

$$= \frac{s^{(2\mu+\nu)}\, \beta^{\nu}\, \Gamma\left(\mu + (\nu/2) + \frac{1}{2} - \lambda - k\right)}{2^{2\mu+\nu-1}\, \Gamma(\nu+1)\, \Gamma_{\times}\left(\mu + (\nu/2) + \frac{1}{2} - \lambda \pm m\right)} \int_0^{\infty} e^{-\frac{1}{2}u}\, u^{-2\lambda - \frac{1}{2} + 2\mu + \nu} \cdot$$

$$\cdot\, W_{2k+\frac{1}{2}, 2m}(u)\, {}_1F_3 \left\{ \begin{matrix} \mu + \nu + \frac{1}{2} - \lambda - k \\ \nu+1,\ \mu + \nu + \frac{1}{2} - \lambda \pm m \end{matrix} ;\ -\beta^2 u^2/4s^2 \right\} dx.$$

Evaluating the integral by a known result [**14**], namely

$$\int_0^{\infty} x^{l-1}\, e^{-\frac{1}{2}x}\, W_{k+\frac{1}{2}, m}(x)\, {}_pF_q \left\{ \begin{matrix} \alpha_1, \ldots, \alpha_p \\ \beta_1, \ldots, \beta_q \end{matrix} ;\ -a^2 x^2 \right\} dx$$

$$= \frac{\Gamma_x\left(l + \frac{1}{2} \pm m\right)}{\Gamma\left(l + \frac{1}{2} - k\right)}\, {}_{p+4}F_{q+2} \left\{ \begin{matrix} \alpha_1, \ldots, \alpha_p, (l/2) + (1/4) \pm (m/2), (l/2) + (3/4) \pm (m/2) \\ \beta_1, \ldots, \beta_q, (l/2) + (1/4) - (k/2), (l/2) + (3/4) - (k/2) \end{matrix} ;\ 4a^2 \right\},$$

under a set of conditions, we get

$$W\left[t\, g(t^2/4);\ 2k+\frac{1}{2},\ 2\lambda,\ 2m\right] = \frac{\beta^{\nu}\, \Gamma\left(\mu + \frac{1}{2} + (\nu/2) - \lambda - k\right) \Gamma_x(2\mu + \nu + 1 - 2\lambda \pm 2m)}{\Gamma(\nu+1)\, \Gamma_{\times}\left(\mu + (\nu/2) + \frac{1}{2} - \lambda \pm m\right) 2^{2\mu+\nu-1}} \cdot$$

$$\cdot \frac{s^{-(2\mu+\nu)}}{\Gamma(2\mu + \nu - 2\lambda + 1 - 2k)}\, {}_2F_2 \left\{ \begin{matrix} \mu + (\nu/2) + 1 - \lambda \pm m \\ \nu + 1,\ \mu + 1 + (\nu/2) - \lambda - k \end{matrix} ;\ -\beta^2/s^2 \right\},$$

provided that $\mathrm{R}(2\mu+\nu-2\lambda+1\pm 2m)>0$. By Duplication Formula [13] we find that

$$W\left[t\,g(t^2/4);\ 2k+\frac{1}{2},\ 2\lambda,\ 2m\right]=$$

$$=\frac{\beta^\nu\,\Gamma_x(\mu+(\nu/2)+1-\lambda\pm m)\,2^{2k-2\lambda+1}}{s^{2\mu+\nu}\,\Gamma(\nu+1)\Gamma(\mu+1+(\nu/2)-\lambda-k)\sqrt{\pi}}\,{}_2F_2\left\{\begin{matrix}\mu+(\nu/2)-\lambda+1\pm m\\ \nu+1,\ \mu+(\nu/2)-\lambda k+1\end{matrix};\ -\beta^2/s^2\right\}.$$

Also, $$\varphi\left[s^2:\ k+\frac{1}{2},\ \lambda,\ m\right]=\frac{\beta^\nu\,\Gamma_x(\mu+1+(\nu/2)-\lambda\pm m)}{s^{2\mu+\nu}\,\Gamma(\nu+1)\,\Gamma(\mu+1+(\nu/2)-\lambda-k)}\,.$$

$$\cdot\,{}_2F_2\left\{\begin{matrix}\mu+1+(\nu/2)-\lambda\pm m\\ \nu+1,\ \mu+1+(\nu/2)-\lambda-k\end{matrix};\ -\beta^2/s^2\right\}.$$

Thus the Theorem is illustrated.

This Theorem can be used to evaluate certain infinite integrals.

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