Riv. Mat. Univ. Parma (2) 8 (1967), 375-384

B. P. PARASHAR (*)

Some Theorems on a Generalised Laplace Transform and Results Involving *H*-Function of Fox. (**)

1. - Introduction. The classical LAPLACE transform

$$\varphi(p) = p \int_0^\infty e^{-px} f(x)\, dx\,, \qquad \text{Re}\, p > 0, \tag{1.1}$$

has been generalised by MAINRA [6] in the form

$$\varphi(p) = p \int_0^\infty e^{-(1/2)px} (px)^{-\lambda-\frac{1}{2}}\, W_{k+\frac{1}{2},m}(px)\, f(x)\, dx\,, \tag{1.2}$$

$$\text{Re}\, p > 0, \qquad \text{Re}\,(\mu+1) > 0, \qquad \text{Re}\,(1+\mu-\lambda\pm m) > 0,$$

where $f(x) = O(x^{\mu})$ for small x. (1.2) reduces to (1.1) when $\lambda = k = -m$. It reduces to MEIJER's form [7] when $\lambda = k$ and to that of VARMA [9] when $\lambda = -m$.

The generalised STIELTJES transform is defined as

$$\varphi(p) = p \int_0^\infty \frac{f(x)}{(x+p)^{\mu}}\, dx\,, \qquad |\arg p| < \pi\,. \tag{1.3}$$

(*) Indirizzo: Govt. College, Mandla (M. P.), India.
(**) Ricevuto: 25-VII-1967.

We shall denote (1.1), (1.2), (1.3) by

$$\varphi(p) \doteqdot f(x)\,, \qquad \varphi(p) \underset{\lambda,k,\,m}{\overset{\mathrm{M}}{=}} f(x)\,, \qquad \varphi(p) \underset{\mu}{\overset{\mathrm{S}}{=}} f(x)\,,$$

respectively.

The object of this Note is to derive some theorems on the transform (1.2) and use them to evaluate some infinite integrals involving the H-function of Fox. The results are believed to be new.

The H-function is defined as

$$(1.4)\quad \begin{cases} H_{p,q}^{m,n}\left[x \,\middle|\, \begin{matrix} (a_1\,, e_1) \ldots (a_p\,, e_p) \\ (b_1\,, f_1) \ldots (b_q\,, f_q) \end{matrix} \right] = \\[2ex] = \dfrac{1}{2\pi i} \displaystyle\int_L \dfrac{\prod_{j=1}^{m} \Gamma(b_j - f_j z) \prod_{j=1}^{n} \Gamma(1 - a_j + e_j z)}{\prod_{j=m+1}^{q} \Gamma(1 - b_j + f_j z) \prod_{j=n+1}^{p} \Gamma(a_j - e_j z)}\, x^z\, \mathrm{d}z\,, \end{cases}$$

where

(i) e's and f's are positive, $0 \leqslant m \leqslant q$, $0 \leqslant n \leqslant p$;

(ii) L runs from $-i\,\infty$ to $i\,\infty$, such that all the poles of $\Gamma(b_j - f_j\, z)$ $(j = 1, \ldots, m)$ are to the right and all the poles of $\Gamma(1 - a_j + e_j\, z)$ $(j = 1, \ldots, n)$ are to the left of L.

Since the H-function embraces many special functions as particular cases, the results we arrive at lead to many old and new results on special functions.

We shall abbreviate (1.4) as

$$H_{p,q}^{m,n}\left[x \,\middle|\, \begin{matrix} (a_p\,, e_p) \\ (b_q\,, f_q) \end{matrix} \right].$$

2. – We shall now find out the images of the H-function under the transforms (1.2) and (1.3) which shall be required in our investigation.

Lemma 1. *If* $\operatorname{Re}\mu > 1$ *and any one of the following sets of conditions holds:*

(A) $d > 0, \qquad |\arg \alpha| < d\,\pi/2\,,$

(B) $d = 0$, α *real and* >0, *and* $\operatorname{Re}(\delta + 1) < 0$,

where

$$d = \sum_{j=1}^{r} f_j + \sum_{j=1}^{s} e_j - \sum_{j=r+1}^{\tau} f_j - \sum_{j=s+1}^{\sigma} e_j\,, \qquad \delta = \frac{\sigma - \tau}{2} + \sum_{j=1}^{\tau} b_j - \sum_{j=1}^{\sigma} a_j\,,$$

then

$$H^{r,s}_{\sigma,\tau}\left[\alpha x \left| \begin{matrix} (a_\sigma\,,\ e_\sigma) \\ (b_\tau\,,\ f_\tau) \end{matrix} \right.\right] \underset{\mu}{\overset{S}{=}} \frac{p^{2-\mu}}{\Gamma(\mu)}\, H^{r+1,s+1}_{\sigma+1,\tau+1}\left[\alpha p \left| \begin{matrix} (0,\ 1), & (a_\sigma,\ e_\sigma) \\ (\mu - 1,\ 1), & (b_\tau,\ f_\tau) \end{matrix} \right.\right]. \tag{2.1}$$

Proof.

$$\int_0^\infty H^{r,s}_{\sigma,\tau}\left[\alpha x \left| \begin{matrix} (a_\sigma\,,\ e_\sigma) \\ (b_\tau\,,\ f_\tau) \end{matrix} \right.\right] \frac{\mathrm{d}x}{(x + p)^\mu} =$$

$$= \frac{1}{2\pi i}\int_0^\infty \left[\int_L \frac{\prod_{j=1}^{r} \Gamma(b_j - f_j z) \prod_{j=1}^{s} \Gamma(1 - a_j + e_j z)}{\prod_{j=r+1}^{\tau} \Gamma(1 - b_j + f_j z) \prod_{j=s+1}^{\sigma} \Gamma(a_j - e_j z)} (\alpha x)^z\, \mathrm{d}z\right] \frac{\mathrm{d}x}{(x + p)^\mu}\,.$$

The x-integral is absolutely convergent if $\operatorname{Re}\mu > 1$. Following the analysis as in Art. (1.19), p. 49 of [1], the z-integral is absolutely convergent if (A) or (B) holds. The change of order of integration is now justified and leads to the required result.

Lemma 2. *If* $\operatorname{Re}(\varrho + \lambda \pm m) < 0$ *and either of the conditions* (A) *or* (B) *of Lemma* 1 *hold, then*

$$\left\{ \begin{aligned} & x^{-\varrho}\, H^{r,s}_{\sigma,\tau}\left[\alpha x \left| \begin{matrix} (a_\sigma\,,\ e_\sigma) \\ (b_\tau\,,\ f_\tau) \end{matrix} \right.\right] \underset{\lambda,k,m}{\overset{M}{=}} \\ & \underset{\lambda,k,m}{\overset{M}{=}}\, p^\varrho\, H^{r,s+2}_{\sigma+2,\tau+1}\left[\frac{\alpha}{p} \left| \begin{matrix} (\varrho + \lambda - m,\ 1), \ (\varrho + \lambda + m,\ 1), \ (a_\sigma\,,\ e_\sigma) \\ (b_\tau\,,\ f_\tau), \ (\varrho + \lambda + k,\ 1) \end{matrix} \right.\right]. \end{aligned} \right. \tag{2.2}$$

The result is obtained by using ([2], (16), p. 216) and following the procedure of Lemma 1.

From Lemma 2 we have as a special case

$$x^{-\varrho}\, H^{r,s}_{\sigma,\tau}\left[\alpha x \,\middle|\, \begin{matrix}(a_\sigma\,,\ e_\sigma)\\ (b_\tau\,,\ f_\tau)\end{matrix}\right] \doteqdot p^{\varrho}\, H^{r,s+1}_{\sigma+1,\tau}\left[\frac{\alpha}{p} \,\middle|\, \begin{matrix}(\varrho,\ 1),\ (a_\sigma\,,\ e_\sigma)\\ (b_\tau\,,\ f_\tau)\end{matrix}\right]. \tag{2.3}$$

3. - Theorem 1. *If*

$$\psi(p) \doteqdot f(x)\,, \tag{i}$$

$$p^{-\lambda-k}\, f\!\left(\frac{1}{p}\right) \underset{\lambda,k,m}{\overset{\mathrm{M}}{=\!=}} \varphi(x)\,, \tag{ii}$$

then

$$\psi(p) = 2p^{k+1}\int_0^\infty x^{-\lambda}\, k_{2m}(2\sqrt{px})\,\varphi(x)\,\mathrm{d}x, \tag{3.1}$$

provided $\operatorname{Re} p > 0$, $\operatorname{Re}(\mu+1) > 0$, $\operatorname{Re}(1+\mu-\lambda\pm m) > 0$, *where* $\varphi(x) = O(x^{\mu})$ *for small* x *and the integral in* (3.1) *is absolutely convergent.*

Proof. From (ii)

$$f(x) = x^{-k-\frac12}\int_0^\infty e^{-t/(2x)}\, t^{-\lambda-\frac12}\, W_{k+\frac12,m}\!\left(\frac{t}{x}\right)\varphi(t)\,\mathrm{d}t\,.$$

Hence

$$\psi(p) = p\int_0^\infty e^{-px}\left[x^{-k-\frac12}\int_0^\infty e^{-t/2x}\, t^{-\lambda-\frac12}\, W_{k+\frac12,m}\left(\frac{t}{x}\right)\varphi(t)\,\mathrm{d}t\right]\mathrm{d}x\,.$$

Interchanging the order of integration, which is permissible under the given conditions and using ([2] (22), p. 217), we have (3.1).

If $\lambda = -m$ and k is changed to $k-(1/2)$ we obtain a result due to Bhonsle [4].

Example. Put

$$\varphi(x) = x^{-\varrho}\, H^{r,s}_{\sigma,\tau}\left[\alpha x \,\middle|\, \begin{matrix}(a_\sigma\,,\ e_\sigma)\\ (b_\tau\,,\ f_\tau)\end{matrix}\right].$$

Using Lemma 2,

$$f(x) = x^{-\varrho-\lambda-k} H^{r,s+2}_{\sigma+2,\tau+1}\left[\alpha x \,\middle|\, \begin{matrix}(\varrho+\lambda-m,\ 1),\ (\varrho+\lambda+m,\ 1),\ (a_\sigma,\ e_\sigma)\\ (b_\tau,\ f_\tau),\ (\varrho+\lambda+k,\ 1)\end{matrix}\right].$$

Hence, using (2.3),

$$\psi(p) = p^{\varrho+\lambda+k} H^{r,s+2}_{\sigma+2,\tau}\left[\frac{\alpha}{p} \,\middle|\, \begin{matrix}(\varrho+\lambda-m,\ 1),\ (\varrho+\lambda+m,\ 1),\ (a_\sigma,\ e_\sigma)\\ (b_\tau,\ f_\tau)\end{matrix}\right].$$

Thus we have finally, after replacing $\varrho+\lambda$ by ν,

$$(3.2)\quad \begin{cases} \displaystyle\int_0^\infty x^{-\nu} K_{2m}(2\sqrt{px})\, H^{r,s}_{\sigma,\tau}\left[\alpha x \,\middle|\, \begin{matrix}(a_\sigma,\ e_\sigma)\\ (b_\tau,\ f_\tau)\end{matrix}\right] dx = \\ \displaystyle = \frac{1}{2} p^{\nu-1} H^{r,s+2}_{\sigma+2,\tau}\left[\frac{\alpha}{p} \,\middle|\, \begin{matrix}(\nu-m,\ 1),\ (\nu+m,\ 1),\ (a_\sigma,\ e_\sigma)\\ (b_\tau,\ f_\tau)\end{matrix}\right], \end{cases}$$

provided conditions of Lemma 1 hold and $\operatorname{Re}\nu < 1 - \frac{1}{2}|\operatorname{Re} m| + \min_{1\le j\le r} \operatorname{Re}\frac{b_j}{f_j}$. If $p=1$ and all $e's$ and $f's$ are equal to 1 we obtain a known result ([**3**], (11), p. 421).

4. - Theorem 2. *If*

(i) $$\psi(p) \doteqdot f(x)\,,$$

(ii) $$p^{2-\nu} f(p) \underset{\lambda,k,m}{\overset{\mathrm{M}}{=}} \varphi(x)\,,$$

then

$$(4.1)\quad \begin{cases} \displaystyle\psi(p) = \frac{\Gamma(\nu-m-\lambda)\,\Gamma(\nu+m-\lambda)}{\Gamma(\nu-\lambda-k)}\, p\,\cdot \\ \displaystyle\cdot \int_0^\infty t^{-\nu}\, {}_2F_1\left[\begin{matrix}\nu+m-\lambda,\ \nu-m-\lambda;\ -p/t\\ \nu-\lambda-k\end{matrix}\right] \varphi(t)\, dt\,, \end{cases}$$

provided $\operatorname{Re}\nu > 0$, $\operatorname{Re}(\nu-\lambda\pm m) > 0$ *and the integral in* (4.1) *is convergent.*

Proof. From (ii)

$$f(x) = x^{\nu-1} \int_0^\infty e^{-xt/2} (xt)^{-\lambda-1/2} W_{k+1/2,m}(xt) \varphi(t) \, dt .$$

Hence

$$\psi(p) = p \int_0^\infty e^{-px} [x^{\nu-1} \int_0^\infty e^{-xt/2} (xt)^{-\lambda-1/2} W_{k+1/2,m}(xt) \varphi(t) \, dt] \, dx .$$

Interchanging the order of integration and using ([2], (16), p. 216), we have,

$$\psi(p) = \frac{\Gamma(\nu+m-\lambda)\,\Gamma(\nu-m-\lambda)}{\Gamma(\nu-\lambda-k)} p.$$

$$\cdot \int_0^\infty \frac{t^{m-\lambda}}{(p+t)^{\nu+m-\lambda}} \, {}_2F_1\left[\begin{matrix}\nu+m-\lambda, \; m-k; \; p/(p+t) \\ \nu-\lambda-k\end{matrix}\right] \varphi(t) \, dt .$$

The result (4.1) is now obtained by using the relation

$${}_2F_1\left[\begin{matrix}a, \; b; \; 1-z \\ a+b+1-c\end{matrix}\right] = z^{-a} \, {}_2F_1\left[\begin{matrix}a, \quad a+1-c; \quad 1-\dfrac{1}{z} \\ a+b+1-c\end{matrix}\right] .$$

If we put $\lambda = - m$ and change k to $k - \frac{1}{2}$ we obtain a result due to Rathie [8].

Example. Put

$$\varphi(x) = x^{-\varrho} H^{r,s}_{\sigma,\tau}\left[\alpha x \,\middle|\, \begin{matrix}(a_\sigma, e_\sigma) \\ (b_\tau, f_\tau)\end{matrix}\right] .$$

Using Lemma 2,

$$f(x) = x^{\varrho+\nu-2} H^{r,s+2}_{\sigma+2,\tau+1}\left[\frac{\alpha}{x} \,\middle|\, \begin{matrix}(\varrho+\lambda-m, 1), \; (\varrho+\lambda+m, 1), \; (a_\sigma, e_\sigma) \\ (b_\tau, f_\tau), \; (\varrho+\lambda+k, 1)\end{matrix}\right] .$$

Hence, using (2.3) and the relation

$$H^{m,n}_{p,q}\left[x \,\middle|\, \begin{matrix}(a_p, e_p) \\ (b_q, f_q)\end{matrix}\right] = H^{n,m}_{q,p}\left[\frac{1}{x} \,\middle|\, \begin{matrix}(1-b_q, f_q) \\ (1-a_p, e_p)\end{matrix}\right] ,$$

we have

$$\psi(p) = p^{2-\varrho-\nu} H^{r+1,s+2}_{\sigma+2,\tau+2}\left[\alpha p \,\middle|\, \begin{matrix} (\varrho+\lambda-m,\ 1),\ (\varrho+\lambda+m,\ 1),\ (a_\sigma,\ e_\sigma) \\ (\varrho+\nu-1,\ 1),\quad (b_\tau,\ f_\tau),\quad (\varrho+\lambda+k,\ 1) \end{matrix}\right].$$

Thus we finally have

$$(4.2)\quad \begin{cases} \displaystyle\int_0^\infty t^{-\varrho-\nu}\, {}_2F_1\left[\begin{matrix} \nu-m-\lambda,\ \nu+m-\lambda;\ -x/t \\ \nu-\lambda-k \end{matrix}\right] H^{r,s}_{\sigma,\tau}\left[\alpha t \,\middle|\, \begin{matrix} (a_\sigma,\ e_\sigma) \\ (b_\tau,\ f_\tau) \end{matrix}\right] \mathrm{d}t = \\ \displaystyle = \frac{\Gamma(\nu-\lambda-k)}{\Gamma(\nu-m-\lambda)\,\Gamma(\nu+m-\lambda)}\, x^{1-\varrho-\nu}\cdot \\ \qquad \cdot H^{r+1,s+2}_{\sigma+2,\tau+2}\left[\alpha x \,\middle|\, \begin{matrix} (\varrho+\lambda-m,\ 1),\ (\varrho+\lambda+m,\ 1),\ (a_\sigma,\ e_\sigma) \\ (\varrho+\nu-1,\ 1),\ (b_\tau,\ f_\tau),\ (\varrho+\lambda+k,\ 1) \end{matrix}\right]. \end{cases}$$

5. – Theorem 3. *If*

(i) $$\psi(p) \doteqdot x^{\mu-2} f(x)\,,$$

(ii) $$f(p) \doteqdot h(x)\,,$$

(iii) $$p^{2-\nu} h(p) \ \underset{\lambda,k,m}{\overset{\mathrm{M}}{=\!=}}\ \varphi(x)\,,$$

then

$$(5.1)\quad \psi(p) = p^{1-\mu}\int_0^\infty x^{-\nu} E(\nu-m-\lambda,\ \nu+m-\lambda,\ \mu:\ \nu-\lambda-k:\ px)\,\varphi(x)\,\mathrm{d}x\,,$$

where E is the MacRobert's E-function, provided $\mathrm{Re}\, p > 0$, $\mathrm{Re}\,\mu > 0$, $\mathrm{Re}\,\nu > 0$, $\mathrm{Re}\,(\nu-\lambda\pm m) > 0$ *and the integral is convergent.*

Proof. By Theorem 2,

$$f(t) = \frac{\Gamma(\nu-m-\lambda)\,\Gamma(\nu+m-\lambda)}{\Gamma(\nu-\lambda-k)}\ t\int_0^\infty x^{-\nu}\, {}_2F_1\left[\begin{matrix} \nu+m-\lambda,\ \nu-m-\lambda;\ -t/x \\ \nu-\lambda-k \end{matrix}\right] \varphi(x)\,\mathrm{d}x\,.$$

Substituting this value of $f(t)$ in the relation

$$\psi(p) = p \int_0^\infty e^{-pt}\, t^{\mu-2} f(t)\, dt$$

and interchanging the order of integration we have

$$\psi(p) = \frac{\Gamma(\nu - m - \lambda)\, \Gamma(\nu + m - \lambda)}{\Gamma(\nu - \lambda - k)}\, p \cdot$$

$$\cdot \int_0^\infty x^{\mu-\nu} \varphi(x) \left\{ \int_0^\infty e^{-pxt}\, t^{\mu-1}\, {}_2F_1\!\left[\begin{matrix} \nu + m - \lambda,\ \nu - m - \lambda;\ -t \\ \nu - \lambda - k \end{matrix}\right] dt \right\} dx\,.$$

Using ([2], (12), p. 299) we finally obtain (5.1).

If $\lambda = -m$ and k is changed to $k - \frac{1}{2}$ we obtain a result due to BHONSLE [5].

Example. Put

$$\varphi(x) = x^{-\varrho}\, H^{r,s}_{\sigma,\tau}\!\left[\alpha x \,\middle|\, \begin{matrix}(a_\sigma\,,\ e_\sigma) \\ (b_\tau\,,\ f_\tau)\end{matrix}\right].$$

Proceeding as in (4.2)

$$x^{\mu-2} f(x) = x^{\mu-\varrho-\nu}\, H^{r+1,s+2}_{\sigma+2,\tau+2}\left[\alpha x \,\middle|\, \begin{matrix}(\varrho + \lambda - m,\ 1),\ (\varrho + \lambda + m,\ 1),\ (a_\sigma\,,\ e_\sigma) \\ (\varrho + \nu - 1,\ 1),\quad (b_\tau\,,\ f_\tau),\quad (\varrho + \lambda + k,\ 1)\end{matrix}\right].$$

(2.3) with Theorem 3 finally leads to

$$(5.2)\quad \begin{cases} \displaystyle\int_0^\infty t^{-\varrho-\nu}\, E(\nu - m - \lambda,\ \nu + m - \lambda,\ \mu\colon\ \nu - \lambda - k\colon\ tx)\, H^{r,s}_{\sigma,\tau}\!\left[\alpha t \,\middle|\, \begin{matrix}(a_\sigma\,,\ e_\sigma) \\ (b_\tau\,,\ f_\tau)\end{matrix}\right] dt = \\ \displaystyle = x^{\varrho+\nu-1}\, H^{r+1,s+3}_{\sigma+3,\tau+2}\!\left[\frac{\alpha}{x} \,\middle|\, \begin{matrix}(\varrho+\nu - \mu,\ 1),\ (\varrho+\lambda - m,\ 1),\ (\varrho+\lambda+m,\ 1),\ (a_\sigma,\ e_\sigma) \\ (\varrho + \nu - 1,\ 1),\ (b_\tau\,,\ f_\tau),\ (\varrho + \lambda + k,\ 1)\end{matrix}\right]. \end{cases}$$

6. – Theorem 4. *If*

(i) $$f(p) \underset{\mu}{\overset{\mathrm{S}}{=}} g(x)\,,$$

(ii) $$p^{2-\lambda-k-\mu}\, g(p) \underset{\lambda,k,m}{\overset{\mathrm{M}}{=}} \varphi(x)\,,$$

then

$$(6.1)\quad f(p) = \frac{\Gamma(k+\mu+m)\,\Gamma(k+\mu-m)}{\Gamma(\mu)}\, p^{k+\frac{1}{2}} \int_0^\infty t^{-\lambda-\frac{1}{2}}\, e^{pt/2}\, W_{\frac{1}{2}-k-\mu,m}(pt)\,\varphi(t)\,\mathrm{d}t\,,$$

provided $\mathrm{Re}\,(k+\mu) > |\,\mathrm{Re}\, m\,|$ *and the integral in* (6.1) *is absolutely convergent.*

Proof. From (ii)

$$g(x) = x^{k+\mu-(3/2)} \int_0^\infty e^{-xt/2}\, t^{-\lambda-\frac{1}{2}}\, W_{k+\frac{1}{2},m}(xt)\,\varphi(t)\,\mathrm{d}t\,.$$

Substituting this value of $g(x)$ in the relation

$$f(p) = p \int_0^\infty \frac{g(x)}{(x+p)^{\mu}}\,\mathrm{d}x\,,$$

interchanging the order of integration and using ([3], (31), p. 237), we obtain (6.1).

Example. Let

$$\varphi(x) = x^{k+\lambda+\mu-2}\, H^{r,s}_{\sigma,\tau}\left[\alpha x \,\middle|\, \begin{matrix}(a_\sigma\,,\ e_\sigma)\\ (b_\tau\,,\ f_\tau)\end{matrix}\right].$$

Using Lemma 2 we have

$$g(x) = H^{s+2,r}_{\tau+1,\sigma+2}\left[\frac{x}{\alpha} \,\middle|\, \begin{matrix}(1-b_\tau\,,\ f_\tau),\ (\mu-1,\ 1)\\ (k+\mu+m-1,\ 1),\ (k+\mu-m-1,\ 1),\ (1-a_\sigma\,,\ e_\sigma)\end{matrix}\right],$$

and using Lemma 1

$$f(p) = \frac{1}{\Gamma(\mu)}\, p^{2-\mu}\, H^{s+2,r+1}_{\tau+1,\sigma+2}\left[\frac{p}{\alpha} \,\middle|\, \begin{matrix}(0,\ 1),\ (1-b_\tau\,,\ f_\tau)\\ (k+\mu+m-1,1),\ (k+\mu-m-1,1),\ (1-a_\sigma\,,e_\sigma)\end{matrix}\right].$$

Thus finally we have

$$(6.2)\quad \begin{cases} \Gamma(k+\mu+m)\,\Gamma(k+\mu-m)\displaystyle\int_0^\infty t^{k+\mu-(5/2)}\, e^{pt/2}\, W_{\frac{1}{2}-k-\mu,m}(pt)\, H^{r,s}_{\sigma,\tau}\left[\alpha t \,\middle|\, \begin{matrix}(a_\sigma\,,\ e_\sigma)\\(b_\tau\,,\ f_\tau)\end{matrix}\right] dt = \\[2ex] = \dfrac{1}{p^{k+\mu-(3/2)}}\, H^{r+1,s+2}_{\sigma+2,\tau+1}\left[\dfrac{\alpha}{p} \,\middle|\, \begin{matrix}(2-k-\mu-m,\ 1),\ (2-k-\mu+m,\ 1),\ (a_\sigma\,,\ e_\sigma)\\(1,\ 1),\ (b_\tau\,,\ f_\tau)\end{matrix}\right]. \end{cases}$$

The author is grateful to Dr. C. B. L. VERMA for guidance in the preparation of this paper.

References.

[1] A. ERDÉLYI, *Higher Transcendental Functions*, Vol. 1, McGraw Hill, New York 1953.

[2] A. ERDÉLYI, *Tables of Integral Transforms*, Vol. 1, McGraw Hill, New York 1954.

[3] A. ERDÉLYI, *Tables of Integral Transforms*, Vol. 2, McGraw Hill, New York 1954.

[4] B. R. BHONSLE, *A theorem involving generalised Laplace integral*, Bull. Calcutta Math. Soc. **48** (1956), 177-180.

[5] B. R. BHONSLE, *On a property of generalised Laplace's transformation*, Bull. Calcutta Math. Soc. **50** (1958), 6-8.

[6] V. P. MAINRA, *A new generalisation of the Laplace transform*. Bull. Calcutta Math. Soc. **53** (1961), 23-31.

[7] C. S. MEIJER, *Eine neue Erweiterung der Laplace Transformation* (I). Nederl. Akad. Wetnsch., Proc. **44** (1941), 727-737.

[8] C. B. RATHIE, *A study of a generalisation of the Laplace's integral*, Proc. Nat. Acad. Sci. India **21** (1952), 231-249.

[9] R. S. VARMA, *On a generalisation of Laplace integral*, Proc.Nat.Acad.Sci.India **20** (1951), 209-216.

* * *