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Inversion Integrals for the Integral Transforms with Bessel Function in the Kernel. (**)

1. – TA LI [2] gave an inversion integral for the integral transform whose kernel is a CHEBYSHEV polynomial. WIDDER [3] used the methods of operational calculus to solve the integral equation whose kernel involves a LAGUERRE polynomial.

In the present paper we solve two integral equations with the BESSEL function $J_\nu(x)$ in the kernel. Our method is somewhat different than the methods used by the above authors.

BESSEL function $J_\nu(x)$ is defined as

$$J_\nu(x) = \sum_{m=0}^{\infty} \frac{(-1)^m (x/2)^{\nu+2m}}{m!\,\Gamma(\nu+m+1)} \,. \tag{1.1}$$

The following results will be used in the sequel. If in [1] (p. 194, (63)) we put $\mu = 1$ and replace a by $z\sqrt{y}$, we get

$$\int_0^y x^{\nu/2} J_\nu(z\sqrt{xy})\,\mathrm{d}x = (2/z)\, y^{\nu/2} J_{\nu+1}(zy), \qquad \nu > -1 \,. \tag{1.2}$$

Similarly from [1] (p. 205, (34)) we have

$$\int_y^\infty x^{-\nu/2} J_\nu(z\sqrt{xy})\,\mathrm{d}x = (2/z)\, y^{-\nu/2} J_{\nu-1}(zy), \qquad \mathrm{Re}\,\nu < 1/2. \tag{1.3}$$

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2. – Theorem I. *If the integral equations*

$$\int_0^x J_\nu(z\sqrt{xy})\, g(y)\, dy = \varphi(x,\, z), \qquad \nu > 1/2\,, \tag{2.1}$$

and

$$\int_0^\infty x^{-\nu/2}\, \varphi(x,\, z)\, dx = \psi(z) \tag{2.2}$$

exist, then the solution of (2.1) *is given by*

$$g(y) = \tfrac{1}{2}\, y^{(\nu/2)+1} \int_0^\infty J_{\nu-1}(zy)\, z^2\, \psi(z)\, dz. \tag{2.3}$$

The solution (2.3) is in the terms of the function $\psi(z)$. But since $\psi(z)$ is in the form of the MELLIN-transform of $\varphi(x,\, z)$ the known function, can be easily evaluated. We suppose that both $\varphi(x,\, z)$ and $\psi(z)$ exist. Putting down the value of $\varphi(x,\, z)$ from (2.1) in (2.2), we have

$$\int_0^\infty x^{-\nu/2} \Big\{ \int_0^x J_\nu(z\sqrt{xy})\, g(y)\, dy \Big\}\, dx = \psi(z).$$

Changing the order of integration which is possible due to the convergence of the integrals involved, we get

$$\int_0^\infty g(y) \Big\{ \int_y^\infty x^{-\nu/2}\, J_\nu(z\sqrt{xy})\, dx \Big\}\, dy = \psi(z).$$

From (1.3) we get following after little adjustment

$$\int_0^\infty J_{\nu-1}(zy)\, \sqrt{zy}\; y^{-(\nu+1)/2}\, f(y)\, dy = \tfrac{1}{2}\, z^{3/2}\, \psi(z).$$

This is in the form of HANKEL-transform, hence inverting this with the help of [1] (p. 5, (1)) we get (2.3).

3. – Theorem II. *If the integral equations*

$$\int_x^\infty J_\nu(z\sqrt{xy})\, g(y)\, dy = \varphi_1(x,\, z), \qquad \nu > -1, \tag{3.1}$$

and

$$\int_0^\infty x^{\nu/2}\,\varphi_1(x,\ z)\,dz = \psi_1(z) \tag{3.2}$$

exist, then the solution of (3.1) *is given by*

$$g(y) = \tfrac{1}{2}\,y^{1-(\nu/2)} \int_0^\infty J_{\nu+1}(yz)\,z^2\,\psi_1(z)\,dz. \tag{3.3}$$

This can be easily proved on the lines of the last theorem making use of the result (2.1).

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References.

[1] A. ERDELYI, *Tables of Integral Transforms*, Vol. 2, Mc Graw-Hill, New York 1954.

[2] TA LI, *A new class of integral transforms*, Proc. Amer. Math. Soc. **11** (1960), 290-298.

[3] D. V. WIDDER, *The inversion of a convolution transform whose kernel is a Laguerre polynomial,* Amer. Math. Monthly **70** (1963), 291-293.

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