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A mathematical model for spinning viscoelastic molten polymers. (**)

Introduction.

In a previous paper [**1**] we described a mathematical model on the melt spinning for newtonian fluids and we showed that this model is acceptable for all materials, as polyesthers, having a negligible elastic component. However, for viscoelastic materials it is generally impossible to neglect the non-newtonian behaviour. In such cases, in fact, both the elastic characteristic of the material and the viscosity dependence on the elongational rate, become too important to be neglected in the spinning process. (See, for example, the experimental work of Paul [**8**]). Therefore it is necessary to formulate again the problem by considering a rheological equation different from the newtonian one.

There are not many papers about this specific argument; on this subject, the Han's work [**2**] is remarkable. This autor investigated the cases of wet spinning and of melt spinning in isothermal conditions, by adopting a three constant Oldroyd model. However with this rheological model, the elongational viscosity is

$$\eta_e = 3\eta_0[1 + \bar{\gamma}(\lambda_0 - \mu_0)] \tag{1}$$

and thus it holds for viscoelastic materials having elongational viscosity increasing with the rate of elongation, for example low density polyethylene and

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high density polyethylene at low elongational rates; on the contrary there are other viscoelastic materials having the elongational viscosity as a decreasing function of the elongational rate, for example polystyrene, polypropylene and high density polyethylene at high elongational rates.

Therefore in order to formulate a spinning model including all the viscoelastic materials, the Oldroyd model must be abandoned and a more general rheological equation used.

At this purpose we consider interesting the work of La Monte and Han [**3**], which deals with the viscoelastic spinning problem by using the following empirical equation for the elongational viscosity

$$\eta_e = 3 \exp(E/RT)[\alpha(k_1 + k_2\,\bar{\gamma}^{q-1})]\,. \tag{2}$$

At the present it is not possible to know from which rheological equation eq. (2) arises.

However if $q = 2$ and $k_1 = 1$, eq.(2) reduces to eq.(1) with $k_2 = \lambda_0 - \mu_0$ and $\eta_0 = \exp(E/RT)$. Therefore we can consider k_1, k_2 and q as elastic parameters and α and E as viscosity parameters.

The aim of this paper is first, to deduce the macroscopic viscoelastic spinning equations and then to study the influence of the material parameters on the spinning process and the yarn birefringence.

In particular our purpose is to discover under which conditions the viscoelastic parameters make the materials spinnable.

Moreover, we are interested in calculating an optimum process for spinning at maximum throughput a yarn of preselected denier and birefringence.

1. - Spinning equations.

We know (1) that the macroscoopic equations of spinning are:

$$\text{mass flow rate:}\quad W = \varrho v_z A \tag{3}$$

$$\text{eq. of motion:}\quad \frac{\partial F}{\partial z} = \frac{\partial W}{\partial t} + \frac{\partial}{\partial z}(v_z W) - A\varrho g + 2\sqrt{\pi A}\,P_f \tag{4}$$

$$\text{eq. of energy:}\quad \varrho C_p\left(\frac{\partial T}{\partial t} + v_z\frac{\partial T}{\partial z}\right) = 2\sqrt{\frac{\eta}{A}}\,h(T_\infty - T)\,. \tag{5}$$

Now we must add an other equation giving the connection between spinning tension and material properties.

Since

(6) $$P_{zz} = \eta_e \bar{\gamma} = \eta_e \frac{\partial v_z}{\partial z} ,$$

we have

(7) $$F(z) = \int_0^{R(z)} 2\pi r P_{zz}\, \mathrm{d}r = \int_0^{R(z)} 2\pi r \eta_e \frac{\partial v_z}{\partial z}\, \mathrm{d}r = \bar{\gamma}\langle\eta_e\rangle A .$$

We now suppose that the average elongational viscosity $\langle\eta_e\rangle$ is given by eq. (2); then eq. (7) becomes

(8) $$F(z) = 3\exp(E/RT)\alpha[k_1 + k_2\bar{\gamma}^{q-1}]A \frac{\partial v_z}{\partial z} .$$

Eq. (8) is the desired equation and it can be used as the macroscopic rheological equation for spinning viscoelastic molten polymers.

We assume, with Y. Ohzava and coworkers [**6**], that the skin friction at z is expressed by:

— for co-current flow

(9) $$P_f = \pm\, 0.353\, \varrho^* \nu^{*0.81} R(z)^{-0.81} |v_z - v_\infty|^{1.19} ,$$

where the positive sign is for $v_z > v_\infty$ and negative sign is for $v_z < v_\infty$;

— for countercurrent flow

(10) $$P_f = 0.353\, \varrho^* \nu^{*0.81} R(z)^{-0.81} (v_z + v_\infty)^{1.19} .$$

For the heat transfer coefficient one has with Kase and Matsuo [**4**]

(11) $$h = 0.473 \times 10^{-4} A^{-0.333} [v_z^2 + (8v_y)^2]^{0.167} .$$

Eqs. (3), (4), (5) and (8) become in a steady-state

(12) $$W = \varrho v_z A = \text{constant} ,$$

(13) $$\frac{\mathrm{d}F}{\mathrm{d}z} = \frac{\mathrm{d}}{\mathrm{d}z}(v_z W) - A\varrho g + 2\sqrt{\pi A}\, P_f ,$$

$$(14) \qquad \varrho C_p v_z \frac{\mathrm{d}T}{\mathrm{d}z} = 2\sqrt{\frac{\pi}{A}}\, h(T_\infty - T)\,,$$

$$(15) \qquad F(z) = 3\exp(E/RT)\alpha\left[k_1 + k_2\left(\frac{\mathrm{d}v_z}{\mathrm{d}z}\right)^{q-1}\right] A\,\frac{\mathrm{d}v_z}{\mathrm{d}z}\,,$$

plus boundary conditions (¹)

$$(16) \qquad A(0) = A_0\,, \quad T(0) = T_0\,, \quad A(L) = A_L\,, \quad T(L) = T_L\,.$$

2. - Solution of the spinning equations.

An analytical solution of the system (11)-(16) is not possible, thus we must choose a numerical method of integration. To this purpose by deriving respect to z the eq. (15) and then expliciting respect to $\mathrm{d}^2v_z/\mathrm{d}z^2$ the resulting expression, we reduce the system (12)-(16) to the following equivalent system

$$(17) \qquad \frac{\mathrm{d}v_z}{\mathrm{d}z} = Y(z)\,,$$

$$(18) \qquad \frac{\mathrm{d}Y}{\mathrm{d}z} = R_2(v_z,\, Y,\, T)\,,$$

$$(19) \qquad \frac{\mathrm{d}T}{\mathrm{d}z} = R_1(v_z,\, T)\,,$$

plus initial conditions

$$(20) \qquad Y(0) = \left(\frac{\mathrm{d}v_z}{\mathrm{d}z}\right)_{z=0} \equiv v_0'\,, \quad v_z(0) = v_0\,, \quad T(0) = T_0\,,$$

where

$$(21) \qquad R_1(v_z,\, T) \equiv \bar{\varrho} v_z^{-0.167}(v_z^2 + \bar{v}_y)^{0.167}(T_\infty - T)\,,$$

(¹) The Barus effect does not permit to take A_0 equal to the extrudate section a_0. In order to calculate A_0 from a_0, the Tanner method [5] can be used: $A_0/a_0 = (1. + 0.5 f_w^2)^{1/3}$; where $f_w = N_{1w}/2\tau_w = (\dot{\gamma}\lambda)_w$ = recoverable shear evaluated at the wall, $N_1 = P_{11} - P_{22}$, $\tau = P_{12}$, λ = relaxation time, $\dot{\gamma}$ = shear rate and w = *at* the spinneret wall.

$$R_2(v_z, Y, T) \equiv \left[v_z W Y - \overline{W} + W_0 v_z^{0.905} |v_z \pm v_\infty|^{1.19} + \right.$$

$$\left. + \left(d \frac{R_1}{T^2} Y + \frac{Y^2}{v_z}\right) \exp(d/T)(\bar{\alpha} + \alpha_0 Y^{q-1})\right] / \exp(d/T)(\bar{\alpha} + \alpha_0 Y^{q-1} q) , \tag{22}$$

with

$$\overline{W} = Wg , \qquad W_0 = \pm 2\pi^{0.905} \left(\frac{W}{\varrho}\right)^{0.095} 0.353 \varrho^* \nu^{*0.81} , \quad \bar{\alpha} = 3k_1 \alpha \frac{W}{\varrho} ,$$

$$\alpha_0 = 3k_2 \alpha \frac{W}{\varrho} , \quad \bar{\varrho} = \frac{2}{\varrho C_p} \sqrt{\frac{\pi \varrho}{W}}\, 0.473 \cdot 10^{-4} \left(\frac{W}{\varrho}\right)^{-0.333} , \qquad d = E/R .$$

Because v_0' is unknown, we must add two other conditions i.e.

$$T(L) = T_L , \qquad v_z(L) = v_L . \tag{23}$$

Here by using a fourth-order Runge Kutta predictor-corrector integration scheme, realised on digital computer, we can obtain the numerical solution of the system (17)-(20), (23). Of course the spinning tension is given by eq. (15), i.e.

$$F(z) = 3 \exp\left(d/T(z)\right) \alpha [k_1 + k_2 Y(z)^{q-1}] A(z) Y(z) . \tag{24}$$

Now it is interesting to investigate the analyticity properties of eq. (18). We see that the denominator in R_2 can be zero if the elongational rate is (2)

$$\bar{\gamma} = \left(-\frac{k_1}{k_2 q}\right)^{1/(q-1)} \equiv \bar{\gamma}_{cr} \qquad (k_2 \neq 0; \quad q \neq 1, 0) . \tag{25}$$

If the values of k_1, k_2 and q give $\bar{\gamma}$ real and finite in eq. (25), the function R_2 will have a singular point, on the z-axis, when $\mathrm{d}v_z(z)/\mathrm{d}z = \bar{\gamma}_{cr}$.

However, we must note that such singularities have been artificially introduced in the model. In fact it is not possible of course to divide by $[\bar{\alpha} + \alpha_0 Y^{q-1} q]$, when this term becomes zero.

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(2) If $k_2 = 0$ or $q = 1$, the problem reduces to newtonian one. Moreover, if $q = 0$ the denominator in R_2 is zero if $k_1 = 0$. In this last case one can easily simplify the system (13-15) to one of the type

$$\frac{\mathrm{d}y_i}{\mathrm{d}z} = f(z, y_1, y_2, y_3) \qquad (i = 1, 2, 3) , \tag{A1}$$

which may be directly integrated by means of the Runge-Kutta method.

Therefore we suppose to be in the neighborhood of a singular point, thus the elengational rate is given by eq. (25) and the system (13)-(15) may be approximated by the following set of differential equations

$$\frac{dF}{dz} = W\bar{\gamma}_{cr} - \overline{W} v_z^{-1} + W_0 v_0^{-0.095} |v_z \pm v_\infty|^{1.19} , \tag{26}$$

$$\frac{dT}{dz} = \bar{\varrho} v^{-0.167} |v_z^2 + \bar{v}_v|^{0.167} (T_\infty - T) , \tag{27}$$

$$v_z^{-1} [\bar{\alpha} + \alpha_0 \bar{\gamma}_{cr}^{q-1}] \bar{\gamma}_{cr} = F \exp(-d/T) . \tag{28}$$

Moreover being

$$\bar{\alpha} + \alpha_0 \bar{\gamma}_{cr}^{q-1} = \bar{\alpha} + \alpha_0 \left(-\frac{\bar{\alpha}}{\alpha_0 q}\right)^{(q-1)/(q-1)} = \bar{\alpha}((q-1)/q) ,$$

we have

$$v_z = \hat{\alpha} F \exp(d/T) , \tag{29}$$

with

$$\hat{\alpha} = \bar{\alpha} \frac{q-1}{q} \bar{\gamma}_{cr} . \tag{30}$$

Using the eq. (29) into eqs. (26) and (27) one obtains

$$\frac{dF}{dz} = S_1(F, T) , \tag{31}$$

$$\frac{dT}{dz} = S_2(F, T) , \tag{32}$$

with

$$S_1(F, T) \equiv W\bar{\gamma}_{cr} - \frac{\overline{W}}{\hat{\alpha}} F \exp(-d/T) + W_0 \left(\frac{F \exp(-d/T)}{\hat{\alpha}}\right)^{0.095} \cdot \tag{33}$$

$$\cdot \left(\frac{\hat{\alpha}}{F} \exp(d/T) \pm v_\infty\right)^{1.19} ,$$

$$(34) \qquad S_2(F, T) \equiv \bar{\varrho} \left(\frac{F \exp(-d/T)}{\hat{a}} \right)^{0.167} \left[\left(\frac{\hat{a}}{F} \exp(d/T) \right)^2 + \bar{v}_y \right]^{0.167} (T_\infty - T) .$$

The system (31)-(32) has not singular points in a spinning process (F is never zero) if $\hat{\alpha} \neq 0$ (3).

In order to join the solution in the singular points with that in non-singular region, it is necessary to calculate $(\mathrm{d}v_z/\mathrm{d}z)$ using Eqs. (31)-(32). This can be achieved by deriving respect to z eq. (29) i.e.

$$(35) \qquad \frac{\mathrm{d}v_z}{\mathrm{d}z} = - \frac{\hat{a}}{F} \exp(d/T) \left(\frac{S_1}{F} + \frac{\mathrm{d}}{T^2} S_2 \right) .$$

3. - Spinnability conditions.

We use the following spinnability criterion:

(a) near the solidification point the elongational rate approches to zero (4);

(b) during the spinning process the spinning tension must be positive and different to zero;

(c) to certify the fibre spinning in an industrial process, the spinning stress must not cause the thread breakdown, i.e. the spinning stress must not exceed the critical tensile stress of the material.

Now let $P_c(T)$ be the experimental critical tensile stress given as a function of the temperature, and let $(A(z), T(z), F(z))$ be the solution for the spinning process, calculated by the mathematical model developed. Then the spinning stress must satisfy the following condition

$$(36) \qquad P_{zz}(z) \equiv \frac{F(z)}{A(z)} < P_c(z)$$

(3) Really, should may be $\hat{a} = 0$ if $k_1 = 0$, $k_2 \neq 0$, $q > 1$. However, in this case the system (13-15) reduces to more simple one like (A1).

(4) In order to remove the numerical singularity which should appear in eq. (18) at $\bar{\gamma} = 0$ for $q < 1$, we multiply, when is $q < 1$, for $Y^{-(q-1)}$ the numerator and denominator in R_2, i.e.

$$37) \qquad \frac{\mathrm{d}Y}{\mathrm{d}z} = \Big[v_z W Y^{1-(q-1)} - \overline{W} Y^{-(q-1)} + W_0 v_z^{0.905} Y^{-(q-1)} \cdot$$

$$\cdot |v_z \pm v_\infty|^{1.19} + \left(d \frac{R_1}{T^2} Y + \frac{Y^2}{v_z} \right) \exp(d/T) (Y^{-(q-1)} \bar{\alpha} + \alpha_0) \Big] / \exp(d/T) (\bar{\alpha} Y^{-(q-1)} + \alpha_0 q) .$$

where $P_c(z)$ is the critical tensile stress along the spinning axis, obtained by resolving the system

$$\begin{cases} P_c = P_c(T) \\ T = T(z) . \end{cases}$$

If the spinning tension is positive for elongational rate values grater than a minimum value different to zero only, the spinnability is hindered. In fact in such cases it will not be possible to arrive at the solidification point with a positive tension.

The situation is summarized in tab. 1.

In the last we note that as the elongational rate increases, a material can assume different viscoelastic parameters values. This be considered in the spinning process calculations.

Because the more general elongational viscosity line is like in fig. 1, we will divide the variability regions in three zones that are possible to fit with the model reported in eq. (2).

Of course a material will be spinnable if its behaviour at the different elongational rates, satisfies the spinnability conditions (tab. 1).

Moreover, it is interesting to note that the newtonian behaviour at low elongational rates promoves the spinnability and in same cases makes possible the spinning process.

For example, let the spinning conditions be

$$W = 0.568 \times 10^{-1}\,(\text{g./sec.})\,, \quad T_0 = 270\,(°\text{C})\,, \quad T_L = 115\,(°\text{C})\,, \quad A_0 = 0.01\,(\text{cm}^2)\,,$$
$$A_L = 0.924 \times 10^{-4}\,(\text{cm}^2)\,, \quad T_\infty = 25\,(°\text{C})\,, \quad v_\infty = 0\,, \quad v_y = 0\,,$$

and let the characteristics of the material be

$$\varrho = 0.9\,(\text{g./cm}^3)\,, \qquad C_p = 0.493\,(\text{cal./g. °C})\,;$$

let the elongational viscosity for $\bar{\gamma} < 0.01\,(\text{sec}^{-1})$ be

$$\eta_e = 3 \times 0.478 \times 10^4 \exp(3000/T) \tag{38}$$

and for $\bar{\gamma} \geqslant 0.01\,(\text{sec}^{-1})$

$$\eta_e = 3 \times 0.139 \exp(3000/T)\,\bar{\gamma}^{-2.27}\,. \tag{39}$$

In this case the mathematical model gives a solution as is shown in fig. 2-(a), i.e. the filament section settles on the prefixed value $A_L = 0.924 \cdot 10^{-4}$ (cm^2) in the neighborhood of the solidification point.

Instead if the elongational viscosity should be given by eq. (39) still for $\bar{\gamma} < 0.01$ (sec^{-1}), the section should not settle on the prefixed value, fig. 2-(b).

In this last case, therefore, it should be more difficult obtain a thread with constant denier, in an industrial process, because little variations in the spinning conditions, always possible, should cause notable variations in the thread section (5). This is well known from pratical industrial experience. For example it is not easy to obtain constant sections for the polypropilenic materials. In fact the polypropylene has not newtonian behaviour at low elongational rates. La Monte and Han observed [**3**] that for this material, q-parameter is approximately 0.117 over the range of elongational rates (0.04 — 0.4) sec.$^{-1}$.

4. - Computer program for calculating the spinning solution.

We have already shown in this paper that to integrate the system (12)-(16) we use the auxiliary system (17)-(20).

In the neighborhood of a singular point this system is replaced by the eqs. (31), (32) and (35).

If $q < 1$, eq. (18) is replaced by the eq. (37).

To certify the fibre spinning in industrial processes it is needed to add the spinnability conditions already seen also. Hence the system for the spinning viscoelastic materials is

A) system (17)-(20)
- a) eq. (18) must be replaced with eq. (37) if $q < 1$,
- b) this system must be replaced by (31), (32) and (35) if there is a singular point in R_2,
- c) other auxiliary systems type ($A1$), if $q = 0$, $k_1 = 0$, or $k_1 = 0$, $q > 1$.

B) $\lim_{z \to L} \bar{\gamma}(z) = 0$. C) $P_{zz}(z) < P_c(z)$. D) $F(z) > 0$.

(5) This agrees with Matovic and Pearson [7] which noted that elongational viscosity as $\eta_e = 3\eta_0 k_2 \bar{\gamma}^{(q-1)}$, ($q < 1$), hinders spinnability.

Of course albeit from the mathematical point of view the conditions B), C) and D) are superabundant respect to the system (17)-(20), they are necessary in order to consider acceptable the spinning solution from the industrial point of view. So if under a set of spinning conditions, the numerical integration of the system (17)-(20) gives a solution which does not satisfy the one among the conditions B), C) and D) at last, we can conclude that the material is not spinnable.

To obtain the solution of the system A), B), C) and D) we can use a digital computer. To this purpose, we elaborated a calculation program, Fortran language, by the which it is possible to rapidly learn the spinning solutions.

The program imput and output are as follows:

Imput: $W, \varrho, C_p, A_0, A_L, T_0, T_L, T_\infty, \eta_e^N, \eta_e, v_\infty, v_y$,

Output: $F_L^N, A^N(z), T^N(z), A(z), T(z), F(z), \Delta n$,

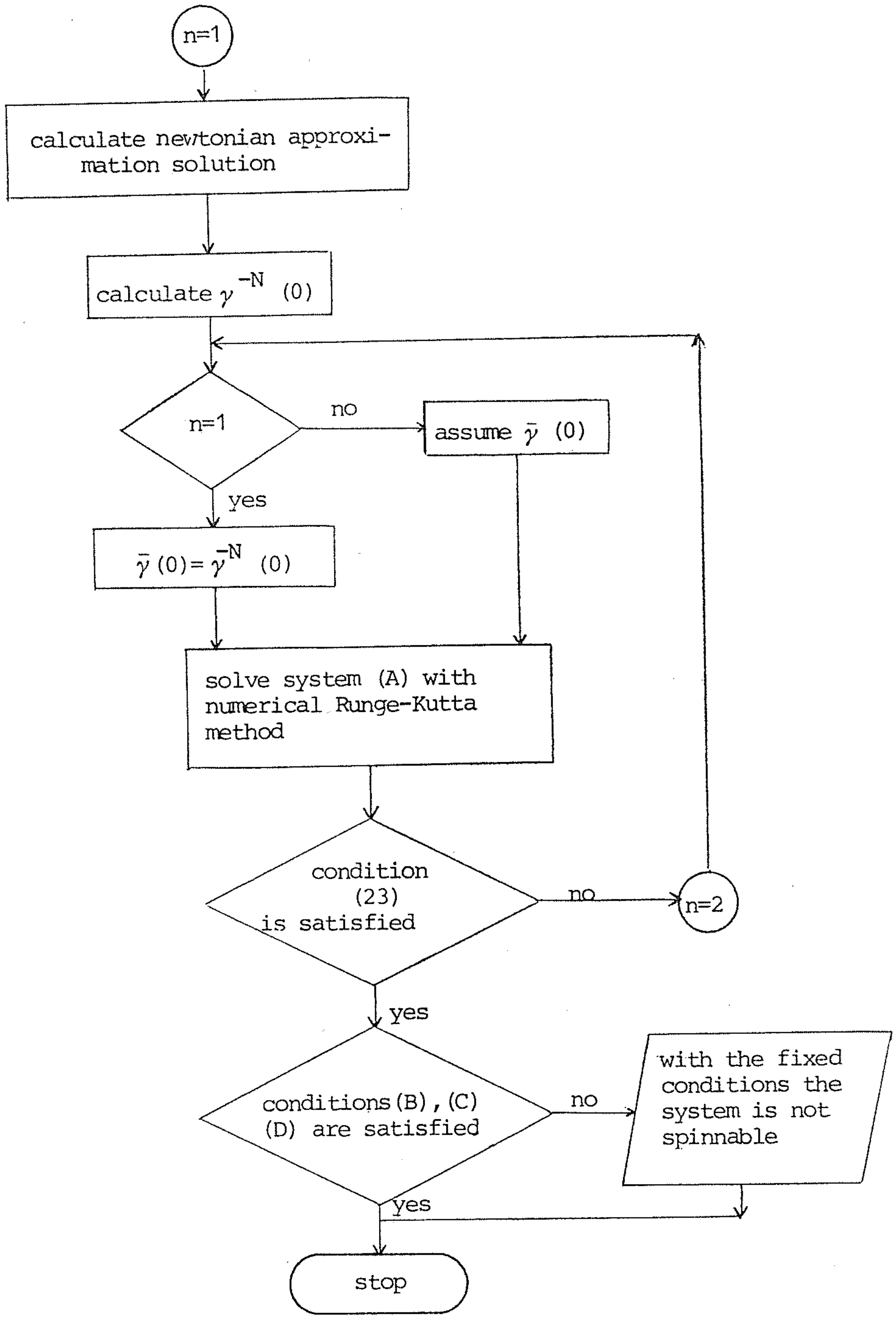
where:

$$\eta_e^N = \text{newtonian limit of the } \eta_e \text{ i.e. } \eta_e^N = \lim_{\bar{\gamma}\to 0} \eta_e(\bar{\gamma})$$

$F_L^N, A^N(z), T^N(z)$ = spinning tension at the solidification point, cross-section and temperature profiles along the spinning axis respectively in the newtonian approximation.

The newtonian approximation is usuful to calculate the first tentative value of $\bar{\gamma}(0)$ in the predictor-corrector scheme used to satisfy the boundary conditions (23).

Details of the computational procedure used are given in the following flow-chart scheme.


n=1
calculate newtonian approximation solution
calculate γ^{-N} (0)
n=1
no
assume $\bar{\gamma}$ (0)
yes
$\bar{\gamma}(0)=\gamma^{-N}(0)$
solve system (A) with numerical Runge-Kutta method
condition (23) is satisfied
no
n=2
yes
conditions (B), (C) (D) are satisfied
no
with the fixed conditions the system is not spinnable
yes
stop

5. - Spinning solution dependence on the material parameters and the condition spinning parameters.

We note that A, T and F have the following functional dependence

$$A = A(z;\, \eta_e,\, \varrho,\, C_p,\, W,\, A_0,\, A_L,\, T_\infty,\, T_0) \tag{40}$$

$$T = T(z;\, \eta_e,\, \varrho,\, C_p,\, W,\, A_0,\, A_L,\, T_\infty,\, T_0) \tag{41}$$

$$F = F(z;\, \eta_e,\, \varrho,\, C_p,\, W,\, A_0,\, A_L,\, T_\infty,\, T_0)\,. \tag{42}$$

The dipendence on the η_e implies that A, T and F are related to the visco-elastic parameters E, α, k_1, k_2 and q.

These dependences can be graphically represented by a digital computer. So one can note that the spinning solution dependence on the parameters ϱ, C_p, W, A_0, A_L, T_∞ and T_0 is similar to the newtonian case (1).

However, now we can obtain a new information, i.e. along the spinning axis the tension is not constant, having a variation caracterized from a depression zone, near the spinneret, more or less accentuated and narrow.

6. - Critical spinning conditions.

By the spinning solutions it is possible to know the fibre birefringence; in fact one has (1)

$$\Delta n = \frac{M}{(273 + T)K}\, \frac{F_L}{A_L}\,. \tag{43}$$

According to eqs. (40)-(42), eq. (43) implies that the birefringence has the following dependence on the spinning parameters

$$\Delta n = \Delta n(\eta_e,\, \varrho,\, C_p,\, W,\, A_0,\, A_L,\, T_\infty,\, T_0)\,. \tag{44}$$

By the aid of the eq. (44) and the spinning solutions, we can obtain the critical spinning condition, i.e. the conditions which realise the maximum fibre production with fixed section and birefringence Δn_0. In fact it is enough to resolve the system

$$W = \text{maximum}\,, \qquad \Delta n(A_0, W) = \Delta n_0\,, \qquad P_{zz}(z;\, A_0, W) < P_c\,. \tag{45}$$

This is well explained in a previous paper [**1**].

The resolution of the system (45) allows us to know the maximum collection rate i.e.

$$v_L^{(\text{maximum})} = W/A_L . \tag{46}$$

This information is desiderable in view of the technological importance of the spinning process in the manufacture of man-made fibres.

Conclusions.

The mathematical model developed, describes the dynamics of non-isothermal melt spinning for the viscoelastic materials. Because a right constitutive equation for all viscoelastic materials is unknown, we used, with La Monte and Han [**3**], an empirical equation for the elongational viscosity. Thus the spinning tension has been related to the empirical viscoelastic parameters (E, α, k_1, k_2, q) and the elongational rate $\bar{\gamma}$.

Then the spinning solution $A(z)$, $T(z)$ and $F(z)$, i.e. the profiles of the cross-sectional area, temperature and spinning tension, along the spinning axis, depends on these viscoelastic parameters (longer than ϱ, C_p, T_∞, T_0, A_0, A_L and W as in the newtonian model). We showed the influence of all these different parameters on the spinning tension along the spinning axis and its shape very much related to the q parameter.

By adopting the following criterion for spinnability:

(a) near the solidification point the elongational rate approches to zero,

(b) the spinning tension is always positive and different to zero,

(c) the tensile stress does not exceed the critical tensile stress of the material

we have discussed the problem of spinnability in terms of viscoelastic parameters (α, k_1, k_2, q).

According to Tab. 1, we can conclude that the class of spinnable viscoelastic materials is limited enough and this agree with the difficulties encountered in the spinning such materials.

Moreover we noted that a newtoniann behaviour at low elongational rates, enhances spinnability.

By using a already known equation, we correlated the yarn birefringence with the spinning solution and therefore with all problem parameters. Here we observe that changing the parameters values, the behaviour of Δn is generally monotone except for q. Of course this is caused by the similar behaviour of the spinning tension.

Index of notations.

A = thread section

$A(z)$ = thread section at the z distance from the spinneret

C_p = specific heat at constant pressure

D = swelled extrudate diameter

d = die diameter

E = constant

F = spinning tension

$F(z)$ = spinning tension at the z distance from the spinneret

g = gravitational accelleration

h = heat transfer coefficient

K = Boltzmann constant = 3.31×10^{-24} cal/°C

k_1, k_2 = constants

L = solidification point on the thread

M = constant determined by optical properties of the molecules

P_c = breakdown

P_{zz} = spinning stress

P_{ij} = stress tensor

P_f = skin friction

q = constant

R = gas constant = 1.987 cal/degree

$R(z)$ = radius of the thread section at the z distance from the spinneret

T = temperature

$T(z)$ = temperature at the z distance from the spinneret

T_∞ = temperature of the air

t = time coordinate

v_∞ = air velocity parallel to the thread

v_y = air velocity perpendicular to the thread

$(v_r, v\varphi\, v_z)$ = rate components in the cylindrical reference system

(r, ϕ, z) = cylindrical reference system

α = constant in the elongational viscosity expression

$\bar{\gamma}$ = elongational rate

η_e = elongational viscosity

η_0 = newtonian viscosity

$$\langle \eta_e \rangle = \frac{1}{A} \int_0^{R(z)} 2\pi r \eta_e \, dr = \text{average elongational viscosity}$$

λ_0, μ_0 = elastic constants

ν^* = air kinematic viscosity

ϱ = density

ϱ^* = air density.

Captions to the figures.

Fig. 1. - Representation of the elongational viscosity vs. elongational rate behaviour. In the regions I, II and III the eq. (2) is a good model to fit the experimental data for the elongational viscosity.

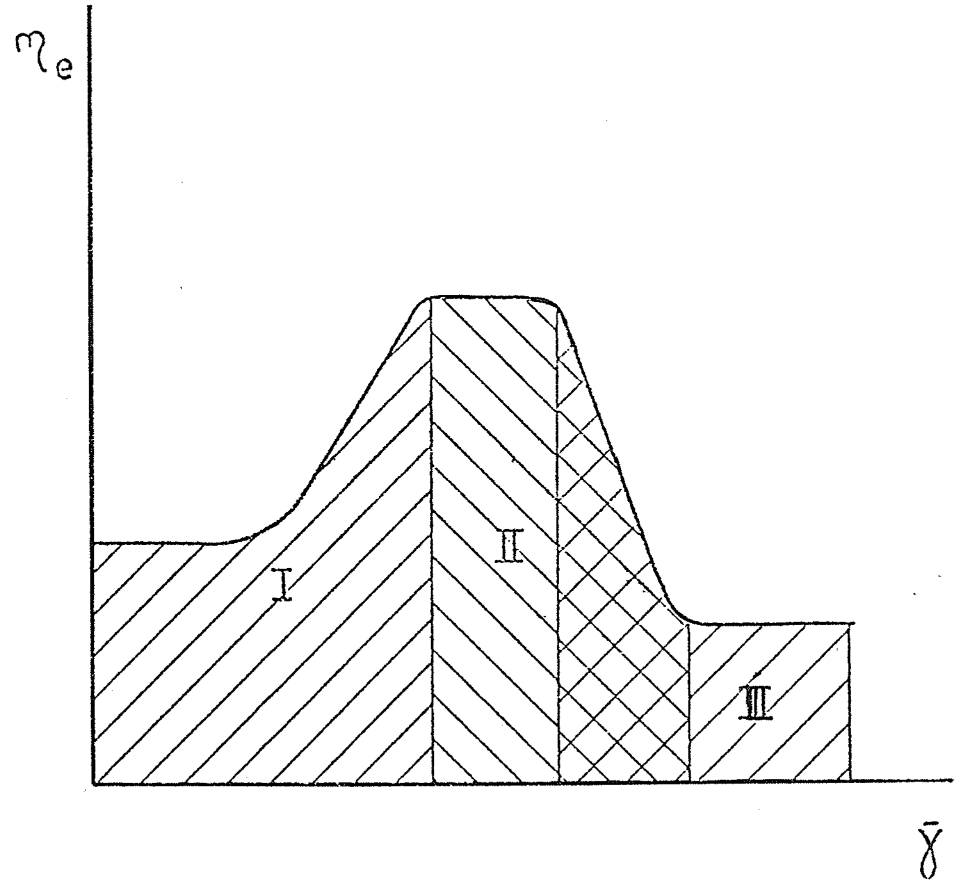


Fig. 1.

Fig. 2. - Example of the newtonian behaviour influence at low elongational rate on the spinnability conditions: (a) profiles of the cross-section $A(z)$ and temperature $T(z)$ along the spinning axis in a material with newtonian behaviour at low elongational rates, (b) profiles of the cross-section $A(z)$ and temperature $T(z)$ along the spinning axis in a material with elongational viscosity decreasing in all range of the elongational rate variability.

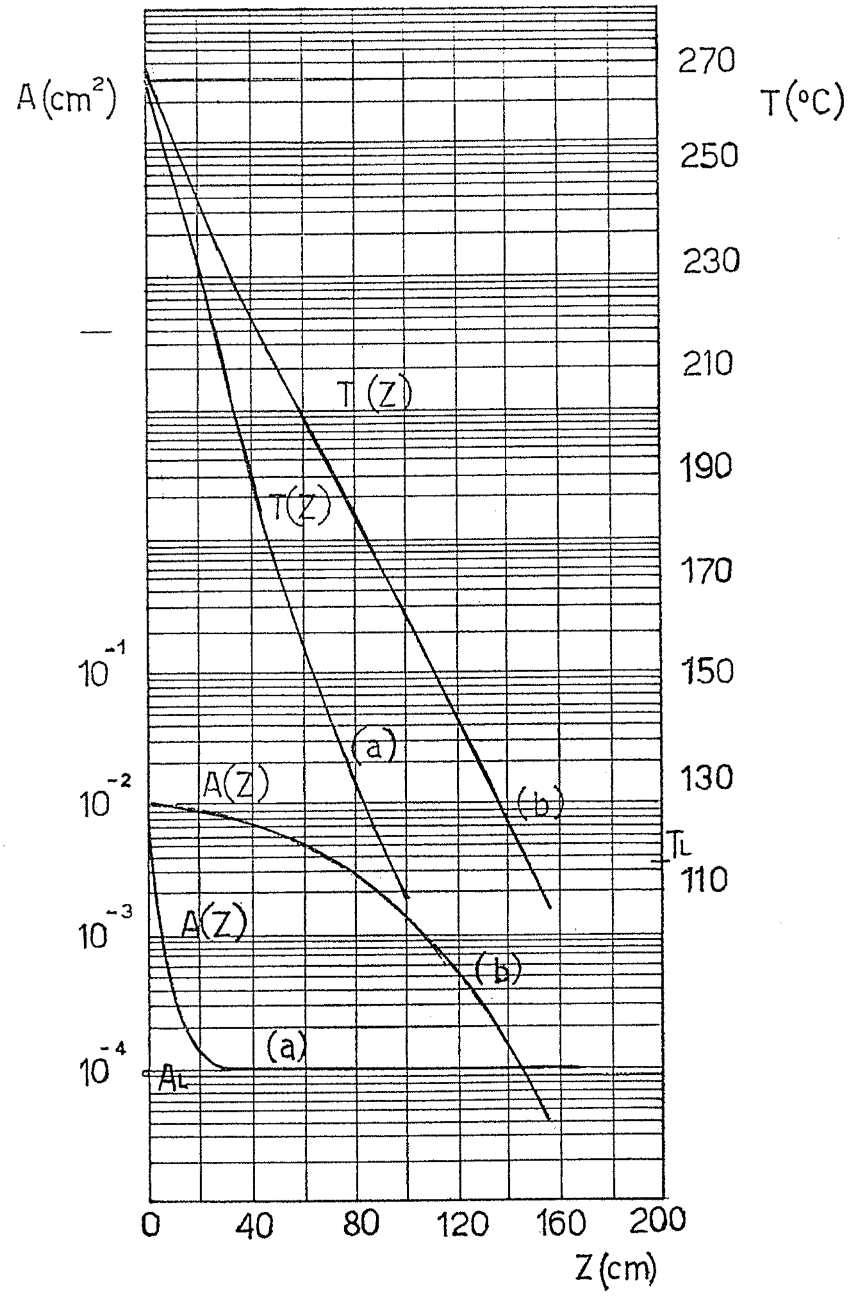


Fig. 2.

TAB. 1. - Viscoelastic parameters and spynnability.

k_1	k_2	k_1/k_2	q	$F \geqslant 0$	singular points in eq. (18)	observations on the spinnability
> 0	$= 0$	∞	—	always	never	newtonian
$= 0$	> 0	$= 0$	> 0	always	$q > 1: \bar{\gamma} = 0$	
					$q < 1$: never	
$= 0$	> 0	$= 0$	$= 0$	always	always	
$= 0$	> 0	$= 0$	< 0	always		
> 0	> 0	> 0	> 0	always	$\bar{\gamma} = b^a$ if $a = \pm 2n$ $n = 1, 2, \dots, \infty$	
> 0	> 0	> 0	$= 0$	always	never	
> 0	> 0	> 0	< 0	always	$\bar{\gamma} = b^a$	
< 0	> 0	< 0	> 0	$q < 1: \bar{\gamma} \leqslant c^a$	$\bar{\gamma} = b^a$	i.m. for $\bar{\gamma} > c^a$
				$q > 1: \bar{\gamma} \geqslant c^a$	$\bar{\gamma} = b^a$	i.m. for $\bar{\gamma} < c^a$
< 0	> 0	< 0	$= 0$	$\bar{\gamma} \leqslant c^{-1}$	never	i.m. for $\bar{\gamma} > c^{-1}$
< 0	> 0	< 0	< 0	$\bar{\gamma} \leqslant c^a$	never	i.m. for $\bar{\gamma} > c^a$
> 0	< 0	< 0	$= 0$	$\bar{\gamma} \geqslant c^{-1}$	never	i.m. for $\bar{\gamma} < c^{-1}$
> 0	< 0	< 0	> 0	$q > 1: \bar{\gamma} \leqslant c^a$	$\bar{\gamma} = b^a$	i.m. for $\bar{\gamma} > c^a$
				$q < 1: \bar{\gamma} \geqslant c^a$	$\bar{\gamma} = b^a$	i.m. for $\bar{\gamma} < c^a$
> 0	< 0	< 0	< 0	$\bar{\gamma} \geqslant c^a$	never	i.m. for $\bar{\gamma} < c^a$

i.m. = inconsistent model

$$a = 1/(q-1)\,, \qquad b = k_1/k_2 q\,, \qquad c = -k_1/k_2\,.$$

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Summary.

A mathematical model for spinning viscoelastic materials is proposed. This work can be considered the continuation of the paper « A mathematical model for spinning molten polymers and conditions of spinning » [**1**], *which treated the case of newtonian materials.*

The viscoelastic system, as more differs from the newtonian as the elastic component is present; thus the viscoelastic mathematical model can not be inferred from the analysis of the previous paper; on the contrary the viscoelastic model includes, as particular case, the newtonian model.

The spinning process was analyzed by addyng the rheological equation for viscoelastic materials to the set of simultaneous partial differential equations describing a general molten spinning process, i.e.

mass flow rate: $$W = \varrho v_z A ,$$

equation of motion: $$\frac{\partial F}{\partial z} = \frac{\partial W}{\partial t} + \frac{\partial}{\partial z}(v_z W) - A\varrho g + 2\sqrt{\pi A}\, P_f$$

equation of energy: $$\varrho C_p \left(\frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial t}\right) = 2\sqrt{\frac{\pi}{A}}\, h(T_\infty - T)$$

rheological equation: $$F = 3 \exp(E/RT)\alpha[k_1 + k_2\bar{\gamma}^{q-1}]A \frac{\partial v_z}{\partial z} .$$

In the above equations, the distance z *from the spinneret and time* t, *are indipendent variables; local velocity* v_z, *cross-sectional area* A, *temperature* T *and spinning tension* F, *are dependent variables;* ϱ *is the density of the polymer filament,* g *the gravitational acceleration,* P_f *the skin friction,* C_p *the isobaric specific heat of the molten filament,* h *the heat transfer coefficient,* T_∞ *the air temperature and* E, α, k_1, k_2 *and* q *are constants;* R *is the gas constant.*

We gave the steady-state ($\partial/\partial t = 0$) *numerical solutions i.e. the filament cross-section* $A(z)$, *filament temperature* $T(z)$ *and filament tension* $F(z)$, *as function of positions* z *and we related them to the parameters which influence the process of spinning: material parameters* (ϱ, C_p, E, α, k_1, k_2, q) *and spinning conditions parameters* (A_L, A_0, T_∞, T_0, W). *We related the yarn birefringence* Δn *to the same parameters also.*

Moreover, we proposed to investigate which bounds impose the spinnability criterion on the viscoelastic parameters (α, k_1, k_2, q) *and which conditions realize the maximum yarn production with fixed denier and section.*

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