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Critical solution for neutron transport in a cylinder with reflector (**)

A GIORGIO SESTINI per il suo 70° compleanno

1. - Introduction

In this work, we study the integral form of the stationary neutron transport equation for an infinitely high cylinder surrounded by a symmetrical reflector of finite thickness, embedded in vacuum.

The natural symmetry of our geometrical scheme allows us to reduce the original tridimensional problem to a monodimensional one. By means of a change of variables, involving the macroscopic cross-section and the average number, we can seek solutions in a fixed functional space for every possible choice of geometrical dimensions.

This space will be $L^2(0, 1)$ as well as $C([0, 1])$; in fact, we shall prove that the solutions in L^2 all belong to C.

Finally, we shall prove that the eigenvalues are continuously and monotonically dependent upon the physical and the geometrical parameters of our problem.

2. - Preliminary remarks

Let R be the radius of the cylinder and h the thickness of the reflector. Following [2], the physical properties are characterized by the total macro-

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scopic cross-section Σ and by the average number of secondaries per collision c. Since we suppose both the cylinder and the reflector to be homogeneous, c will be a two valued step function. For mathematical simplicity, we assume that Σ is constant in all the system.

We also assume that neutrons are monoenergetic, and the scattering and the fission emission are both isotropic.

As the system is embedded in vacuum and we suppose that there are no source of neutrons, the equation is linear [**2**]. In fact, after normalization, we can write

$$\varphi(x) = \int_0^1 [yc(y)T(x, y)]\varphi(y)\,\mathrm{d}y = T\varphi(x)\,, \tag{1}$$

where

$$c(y) = \begin{matrix} c_1 \geqslant 1\,, & 0 \leqslant y \leqslant R/(R+h) \\ c_2 < 1\,, & R/(R+h) < y \leqslant 1\,, \end{matrix}$$

(c_1 is the average number of secondaries in the cylinder, c_2 in the reflector), and

$$4\pi T(x, y) = \int_0^{2\pi}\int_{-\infty}^{+\infty} \frac{\exp[-\sigma(x^2+y^2-2xy\cos\theta+z^2)^{\frac{1}{2}}]}{x^2+y^2-2xy\cos\theta+z^2}\,\mathrm{d}\theta\,\mathrm{d}z$$

$$(\sigma = \Sigma(R+h)).$$

In order to find the solution of (1), we shall study the equation

$$\lambda\varphi = T\varphi\,,$$

i. e. the eigenvalue problem for the operator T, and then we shall discuss the auxiliary equation

$$\lambda(\Sigma, R, h, c_1, c_2) = 1\,.$$

3. - The transport operator

To study the operator T, we start with the following inequality (for a proof, see the Appendix)

$$(x+y)\,T(x, y) < \sigma K_0(\sigma|x-y|) \qquad (x \neq y)\,, \tag{2}$$

where K_0 is the modified Bessel function of zero order [**1**], [**9**].

Since $K_0(z) \sim -\ln z$ as $z \to 0$, [1], (2) ensures that the kernel $[yc(y)\,T(x, y)]$ is square integrable in $(0, 1)\times(0, 1)$, so that the operator $T\colon L^2(0, 1) \to L^2(0, 1)$, is completely continuous [8].

But we can show that $T\colon C([0, 1]) \to C([0, 1])$, and it is completely continuous as well; to see this, it is enough to verify the following statements [4], [5]: (i) for every $x \in [0, 1]$, $[yc(y)\,T(x, y)]$ is measurable with respect to y; (ii) for every $x \in [0, 1]$

$$\lim_{t\to 0} \int_0^1 yc(y)\,|T(x + t, y) - T(x, y)|\,\mathrm{d}y = 0 .$$

Proof. (i) follows at once from (2). To verify (ii), we can apply Lebesgue's theorem on dominated convergence, because of (2). Then, we have to show that

$$\lim_{t\to 0} |T(x + t, y) - T(x, y)| = 0 \qquad \text{a.e. in } [0, 1]\times[0, 1] .$$

But, for $x \neq y$, and t in a suitable closed neighbourhood of zero, the integral, which defines $T(x, y)$, is uniformely convergent in t and the integrand function is continuous. Hence, the limit is zero if $x \neq y$, i.e. almost everywhere.

We now state the first important property of the operator T.

Theorem 1. *The operator T has a unique positive (continuous) eigenfunction. This eigenfunction corresponds to a simple positive eigenvalue, larger in absolute value than all the other eigenvalues of T.*

Proof. Theorem 1 is only a trivial application of a modified statement of Jentzsch's theorem, due to M. G. Krein and M. A. Rutman [4], [5]. We only have to show that T is of positive type, i.e. $Tf(x) > 0$ if $f(x) \geqslant 0$, f not identically zero. In fact,

$$T(x, y) > \sigma/2 \int_{-\infty}^{+\infty} \frac{\exp[-\sigma(4 + z^2)^{\frac{1}{2}}]}{4 + z^2}\,\mathrm{d}z = I\,, \qquad \text{so that}$$

$$Tf(x) > Ic_2 \int_0^1 yf(y)\,\mathrm{d}y > 0 .$$

In the previous discussion, we have stated that T is completely continuous both in $L^2(0, 1)$ and $C([0, 1])$. Obviously, the set of continuous eigenfunctions is a subset of the set of L^2-eigenfunctions. In fact, it is not a proper subset, because we have the

Theorem 2. *If $\varphi \in L^2(0,1)$ is an eigenfunction of T, then φ is continuous.*

Proof. We start by showing that T maps L^2-functions into bounded functions

$$|Tf(x)|^2 = \left|\int_0^1 yc(y)\,T(x,y)\,f(y)\,\mathrm{d}y\right|^2$$

$$\leqslant \|f\|^2_{L^2} c_1^2 \sigma^2 \int_0^1 K_0^2(\sigma|x-y|)\,\mathrm{d}y\,,$$

because of (2). Now, since $K_0(z)\,z^\alpha \to 0$ as $z \to 0$, $\alpha > 0$, we have $K_0^2(\sigma|x-y|) \leqslant C + \sigma^{-2\alpha}|x-y|^{-2\alpha}$, with $0<\alpha<\frac{1}{2}$, and so

$$|Tf(x)|^2 < A + B\,\frac{x^{1-2\alpha} + (1-x)^{1-2\alpha}}{1-2\alpha}\,,$$

where A, B, and C are sufficiently large constants. Hence, if φ is an eigenfunction of T, φ is bounded. Moreover, φ is the image of the bounded function $[\lambda^{-1}\varphi(x)\,c(x)]$ through the operator of kernel $[yT(x,y)]$ (λ is the eigenvalue of φ). $[yT(x,y)]$ is a kernel with a weak singularity and maps bounded functions into continuous ones [**6**].

We conclude this section showing that the operator T is continuous with respect to each of the parameters Σ, R, h, c_1, c_2.

Let T', T'' two operator corresponding to two different sets of parameters. Then,

$$\|T' - T''\|^2 \leqslant \int_0^1\int_0^1 y^2\big(c'(y)\,T'(x,y) - c''(y)\,T''(x,y)\big)^2\,\mathrm{d}x\,\mathrm{d}y\,.$$

That T is continuous with respect to $\sigma = \Sigma(R+h)$ is easily seen by means of Lebesgue's theorem. This ensures the continuity with respect to Σ. The same may be done for R and h; in fact, $T(x,y)$ depends on R (on h) through σ only, and, if c_1 and c_2 are fixed, the step functions $c'(y)$ and $c''(y)$ corresponding to R' and R'' (h', h'') differ only on a set of measure that approaches zero as $R' \to R''$ ($h' \to h''$). The continuity with respect to c_1 and c_2 is trivial.

4. - The symmetric operator

We want now to state some further properties of the spectrum of T. To this aim, we introduce a symmetric operator S defined by the kernel

$$\left[(xy)^{\frac{1}{2}}\big(c(x)\,c(y)\big)^{\frac{1}{2}}\,T(x,y)\right].$$

It is easy to see that S is a completely continuous operator on $L^2(0,1)$, because of (2).

Moreover, the same procedure ensuring continuity of T with respect to the parameters, can be used for S.

The usefulness of S lies on the fact that S and T have the same spectrum.

In fact, if $\lambda\varphi(x) = T\varphi(x)$, $f(x) = (xc(x))^{\frac{1}{2}}\varphi(x)$ is square integrable, and $\lambda f(x) = Sf(x)$. On the contrary, if $\lambda f(x) = Sf(x)$, we have to show that $(xc(x))^{-\frac{1}{2}}f(x)$ is square integrable.

As we proved for T in Theorem 2, we can easily show that $Sf(x)$ is bounded, and so $f(x)$ is bounded too. Moreover,

$$\varphi(x) = f(x)(xc(x))^{-\frac{1}{2}} = \int_0^1 \lambda^{-1} T(x,y)(yc(y))^{\frac{1}{2}} f(y)\,\mathrm{d}y \leqslant M \int_0^1 K_0(\sigma|x-y|)\,y^{-\frac{1}{2}}\,\mathrm{d}y\,,$$

where M is a suitable constant.

Now, using Schwarz inequality with $K_0 \in L^3$ and $y^{-\frac{1}{2}} \in L^{\frac{3}{2}}$, we have

$$|f(x)(xc(x))^{-1/2}| \leqslant M4^{2/3}\Big(\int_0^1 K_0(\sigma|x-y|)^3\,\mathrm{d}y\Big)^{1/3}$$

and, if $0<\alpha<1/3$, then

$$|\varphi(x)| \leqslant A + B\Big(\frac{x^{1-3\alpha}+(1-x)^{1-3\alpha}}{1-3\alpha}\Big)^{1/3},$$

with A, B constants.

A first consequence of this equivalence is the continuity of the eigenvalues of T upon parameters.

Theorem 3. *The eigenvalues of T (of S) are continuous functions of Σ, R, h, c_1, c_2.*

Proof. Such a result follows from the continuity and the symmetry of S. In fact, see [**8**], $|\lambda'_n - \lambda''_n| \leqslant \|S' - S''\|$, $(n = 1, 2, \ldots)$. The following further properties can be established

Theorem 4. *The eigenvalues of T are a countably infinite set of positive numbers, and the first eigenvalue is smaller than c_1 for every Γ, R, h, c_2.*

Proof. Theorem 4 is a direct consequence of the two following properties: (i) S is positive defined, i.e. $(Sf, f) > 0$ if $f \neq 0$; (ii) $\|S\| < c_1$.

To show (i), we use the following integral formula, which will be proved in the Appendix

$$T(x, y) = \int_0^{+\infty} \sigma \tan^{-1}(t/\sigma) J_0(xt) J_0(yt)\, dt ,$$

where J_0 is the Bessel function of zero order.

Now, let $f \in L^2(0, 1)$, $\|f\| = 1$,

$$\begin{aligned}(Sf, f) &= \int_0^1 \int_0^1 (xy)^{\frac{1}{2}} (c(x)c(y))^{\frac{1}{2}} \Big(\int_0^{+\infty} \sigma \tan^{-1}(t/\sigma) J_0(xt) J_0(yt)\, dt\Big) f(x) f(y)\, dx\, dy \\ &= \int_0^{+\infty} \sigma \tan^{-1}(t/\sigma) t^{-1} \Big(\int_0^1 (xt)^{\frac{1}{2}} J_0(xt) c(x)^{\frac{1}{2}} f(x)\, dx\Big)^2 dt .\end{aligned}$$

The function in parentheses is the Hankel transform of the function $\big(c(x)^{\frac{1}{2}} g(x)\big)$, where

$$g(x) = f(x) , \quad 0 \leqslant x \leqslant 1 ; \quad g(x) = 0 , \quad x > 1 .$$

So it follows that $(Sf, f) > 0$ if f is not identically zero.

It remains to justify the interchange of the order of integration. To this aim, it suffice to show that: if $A > 0$, then

$$\Big| \int_A^{+\infty} \tan^{-1}(t/\sigma) J_0(xt) J_0(yt) (xy)^{\frac{1}{2}}\, dt \Big| = I(x, y)$$

is dominated by an integrable function of (x, y) in $(0, 1) \times (0, 1)$. In the Appendix, we have proved that

$$\int_0^{+\infty} \tan^{-1}(t/\sigma) J_0(xt) J_0(yt)\, dt = \frac{1}{2\pi} \int_0^{2\pi} \int_{-\infty}^{+\infty} \tan^{-1}(t/\sigma) J_0(\omega t)\, d\theta\, dt ,$$

where $\omega^2 = x^2 + y^2 - 2xy \cos\theta$.

Integrating by parts, we have

$$I(x, y) \leqslant M \int_0^{\pi/2} \frac{d\theta}{\big(1 - (4xy/(x+y)^2 \operatorname{sen}^2\theta)\big)^{\frac{1}{2}}} ,$$

where the elliptic integral has a logarithmical singularity for $x = y$, [3], and so it is integrable.

As far as (ii) is concerned, since $\sigma \tan^{-1}(t/\sigma)t^{-1} < 1$, we have $(Sf, f) \leqslant c_1 \int_0^{+\infty} G^2(t)\,\mathrm{d}t$, where $G(t)$ is the Hankel transform of $g(x)$. As $g(x) \in L^2(0, +\infty)$ and $\|g\|_{L^2(0,+\infty)} = \|f\|_{L^2(0,1)} = 1$, and the relation between the Hankel transform and the bidimensional Fourier transform, [7], gives $\|G\| = \|g\|$, so $\|S\| \leqslant c_1$.

Moreover, the preceding inequality holds in a strict sense. In fact, by a simple calculation, and using some properties of Hankel transform, we have

$$\|S\| \leqslant c_1 - (c_1 - c_2) \int_{R/(R+h)}^{1} \varphi^2(x)\,\mathrm{d}x ,$$

where φ is the first eigenfunction of S (of T) and

$$\int_{R/(R+h)}^{1} \varphi^2(x)\,\mathrm{d}x > 0$$

according to Theorem 1. This concludes the proof of Theorem 4.

5. - The stationary solution

We can now state the existence of stationary solutions.

In fact, we have $\lim_{c_1 \to +\infty} \lambda_1 = +\infty$, and $\lambda_1 < c_1$, so that, if $c_1 = 1$, then $\lambda_1 < 1$. Then Theorem 3 ensures the existence of a solution of (1), because, for every Σ, R, h, c_2, there exists a $c_1 \in (1, +\infty)$ such that $\lambda_1 = 1$.

But, a more interesting result can be proved.

Theorem 5. *The first eigenvalue is an increasing function of the parameters Σ, R, h, c_1, and c_2.*

Proof. Consider first $\sigma = \Sigma(R + h)$.

Let $\sigma > \sigma'$ and $S = S(\sigma)$, $S' = S(\sigma')$ the respective symmetric operators. Then $(S - S')$ is positive definite. In fact, we have

$$\big((S - S')f, f\big) = \int_0^{+\infty} \big(\sigma \tan^{-1}(t/\sigma) - \sigma' \tan^{-1}(t/\sigma')\big)\, t^{-1} F^2(t)\,\mathrm{d}t ,$$

where $F(t)$ is the Hankel transform of $\big(c(x)\big)^{\frac{1}{2}} g(x)$ as in Theorem 4. Since $\sigma \tan^{-1}(t/\sigma) > \sigma' \tan^{-1}(t/\sigma')$, we conclude that $\big((S - S')f, f\big) > 0$, if f is not identically zero. So it follows

$$\lambda_1(\sigma) - \lambda_1(\sigma') \geqslant (S\varphi', \varphi') - (S'\varphi', \varphi') = \big((S - S')\varphi', \varphi'\big) > 0 ,$$

where φ' is the first eigenfunction of S'. (We note that also $\lambda_n(\sigma) \geqslant \lambda_n(\sigma')$, $n = 2, \ldots$, [8]). Hence, λ_1 is a strictly increasing function of σ.

Assume now that $c_1 > c_1'$, (Σ, R, h, c_2 are fixed) than we have

$$\lambda_1(c_1) \geqslant \lambda_1(c_1') + \int_0^{R/R+h} \int_0^{R/R+h} (xy)^{\frac{1}{2}} T(x, y)(c_1 - c_1')\varphi'(x)\varphi'(y)\,\mathrm{d}x\,\mathrm{d}y$$

$$+ 2 \int_0^{R/R+h} \int_{R/R+h}^{1} (xy)^{\frac{1}{2}} T(x, y)\big((c_1 c_2)^{\frac{1}{2}} - (c_1' c_2)^{\frac{1}{2}}\big)\varphi'(x)\varphi'(y)\,\mathrm{d}x\,\mathrm{d}y\,,$$

it follows that $\lambda_1(c_1) > \lambda_1(c_1')$, because of Theorem 1. The same property obviously holds for c_2. Finally, let $h > h'$ (the other parameters are fixed) and

$$f(x) = \begin{array}{ll} (s/s')^{\frac{1}{2}}\varphi'(sx/s')\,, & 0 \leqslant x \leqslant s'/s\,, \\ 0\,, & s'/s < x \leqslant 1\,. \end{array}$$

with $s = R + h$, and $s' = R + h'$.

Due to Theorem 1, $\lambda_1 > (Sf, f)$. On the other hand, by a simple change of variables, we have $(Sf, f) = (S'\varphi', \varphi')$, and so $\lambda_1(h) > \lambda_1(h')$. The same procedure can be used for R. In this case, f is defined as before with $s = R + h$ and $s' = R' + h$, and we have $(Sf, f) > (S'\varphi', \varphi')$.

As an immediate consequence, we have

Theorem 6. *For every choice of four of the five parameters, the equation $\lambda_1 = 1$ implicitly defines one and only one value of the fifth parameter, that solves the stationary problem.*

So, every parameter is a continuous decreasing function of the others.

Appendix

I. – To show inequality (2), we let $x^2 + y^2 - 2xy\cos\theta + z^2 = \omega^2 + z^2$,

$$I(z) = \int_0^{2\pi} \frac{\exp\left[-\sigma(\omega^2 + z^2)^{\frac{1}{2}}\right]}{\omega^2 + z^2}\,\mathrm{d}\theta \leqslant \exp\left[-\sigma\big((x-y)^2 + z^2\big)^{\frac{1}{2}}\right] \int_0^{2\pi} \frac{\mathrm{d}\theta}{\omega^2 + z^2}$$

$$\leqslant \frac{2\pi \exp\left[-\sigma\big((x-y)^2 + z^2\big)^{\frac{1}{2}}\right]}{\big((x-y)^2 + z^2\big)^{\frac{1}{2}}\big((x+y)^2 + z^2\big)^{\frac{1}{2}}}\,.$$

Hence, $\quad I(z) \leqslant \dfrac{2\pi}{x+y} \cdot \dfrac{\exp\left[-\sigma\big((x-y)^2 + z^2\big)^{\frac{1}{2}}\right]}{\big((x-y)^2 + z^2\big)^{\frac{1}{2}}}\,. \quad$ But

$$\int_{-\infty}^{+\infty} \frac{\exp\left[-\sigma\left((x-y)^2+z^2\right)^{\frac{1}{2}}\right]}{\left((x-y)^2+z^2\right)^{\frac{1}{2}}}\,dz = 2\int_0^{+\infty}\exp\left[-\sigma|x-y|\cosh t\right]dt = 2K_0(\sigma|x-y|),$$

[1], and so $T(x, y) < \dfrac{\sigma}{x+y} K_0(\sigma|x-y|)$.

II. – A more involved proof is needed for the formula at the beginning of Theorem 4.

We start by showing that, if $a > 0$, then

$$\frac{1}{a}\int_a^{+\infty} K_0(z)\,dz = \int_0^{+\infty} \tan^{-1} t J_0(at)\,dt\ .$$

By using the Hankel transform, [7], we have $K_0(z) = \int_0^{+\infty} uJ_0(u)(z^2+u^2)^{-1}\,du$ and so, if $0 < a < B < +\infty$, then

$$\int_a^B K_0(z)\,dz = \int_a^B dz \int_0^{+\infty} \frac{J_0(u)u}{z^2+u^2}\,du = \int_0^{+\infty} J_0(u)\{\tan^{-1}(z/u)|_a^B\}\,du$$

$$= \int_0^{+\infty} J_0(u)\tan^{-1}(u/a)\,du - \int_0^{+\infty} J_0(u)\tan^{-1}(u/B)\,du\ .$$

(Note that the above integrals exist, because $\int_0^{+\infty} J_0(u)\,du = 1$).

It remains to prove that $\lim_{B\to+\infty}\int_0^{+\infty} J_0(u)\tan^{-1}(u/B)\,du = 0$. Now, if $u > A > 0$, $J_0(u) = (2/\pi u)^{\frac{1}{2}}(\cos(u-\pi/4) + f(u))$ where $|f(u)| \leqslant M/u$, M constant, [1], and so

$$\int_0^{+\infty} J_0(u)\tan^{-1}(u/B)\,du = \int_0^A J_0(u)\tan^{-1}(u/B)\,du$$

$$+ \int_A^{+\infty} \tan^{-1}(u/B)\left(\frac{2}{\pi u}\right)^{\frac{1}{2}}\cos(u-\pi/4)\,du + \int_A^{+\infty}\tan^{-1}(u/B)\left(\frac{2}{\pi u}\right)^{\frac{1}{2}} f(u)\,du\ ,$$

where the first integral is obviously convergent to zero as $B \to +\infty$; as far as the third integral is concerned, Lebesgue's theorem can be used. Finally, the second integral is equal to

$$\tan^{-1}(u/B)\left(\frac{2}{\pi u}\right)^{\frac{1}{2}}\operatorname{sen}(u-\pi/4)\Big|_A^{+\infty} + \int_A^{+\infty}\frac{\tan^{-1}(u/B)}{(2\pi)^{\frac{1}{2}}u^{\frac{3}{2}}}\operatorname{sen}(u-\pi/4)\,du$$

$$- \int_A^{+\infty}\left(\frac{2}{\pi u}\right)^{\frac{1}{2}}\operatorname{sen}(u-\pi/4)\,\frac{B}{B^2+u^2}\,du\ ,$$

where the first term approaches zero as $B \to +\infty$; for the second, Lebesgue's theorem is used again. The same theorem can be used for the third, because $\max B/(B^2+u^2) = 1/2u$. Now, by using the integral expression of K_0, we can write

$$\frac{2}{\omega}\int_{\sigma\omega}^{+\infty} K_0(z)\,\mathrm{d}z = \int_{-\infty}^{+\infty} \frac{\exp[-\sigma(\omega^2+z^2)^{\frac{1}{2}}]}{\omega^2+z^2}\,\mathrm{d}z = 2\sigma\int_0^{+\infty}\tan^{-1} t J_0(\sigma\omega t)\,\mathrm{d}t\,.$$

It remains to integrate with respect to θ in $(0, 2\pi)$, and the order of integration has to be interchanged. To this aim, we look for a dominant integrable function, using asymptotic expression of J_0, just as in the above calculation. Then Lebesgue's theorem can be used again. Thus, we obtain

$$4\pi T(x, y) = 2\sigma\int_0^{+\infty}\tan^{-1}(t/\sigma)\{\int_0^{2\pi} J_0(\omega t)\,\mathrm{d}\theta\}\,\mathrm{d}t\,.$$

Since $J_0(\omega t)$ can be expressed using Neumann's addition theorem [**9**], for $x \neq y$,

$$J_0(\omega t) = J_0(xt)J_0(yt) + 2\sum_{n=1}^{\infty} J_n(xt)J_n(yt)\cos(n\theta),$$

and

$$J_n(z) \sim \frac{1}{(2\pi n)^{\frac{1}{2}}}\left(\frac{\mathrm{e}z}{2n}\right)^n \qquad \text{as } n \to \infty\,,$$

we can integrate term by term, and so

$$T(x, y) = \sigma\int_0^{+\infty}\tan^{-1}(t/\sigma)J_0(xt)J_0(yt)\,\mathrm{d}t\,.$$

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Sommario

Si considera l'equazione integrale stazionaria per il trasporto di neutroni nel caso di un cilindro di altezza infinita con riflettore. La simmetria del sistema consente di ridurre il problema al caso unidimensionale. L'operatore di trasporto è lineare, a nucleo non simmetrico. Questo operatore è completamente continuo, sia in C che in L^2. Si dimostra inoltre l'equivalenza delle soluzioni in questi due spazi funzionali. Infine, si stabilisce la dipendenza monotona di ogni parametro, sia fisico che geometrico, rispetto ad ogni altro parametro caratterizzante il nostro problema.

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