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# Acceleration and thermal waves in kinetic theory (**)

## 1 - Introduction

It is well known that the parabolic character of the Fourier equation gives rise to instantaneous heat propagation. This fact, already surprising in a classical framework , is completely unacceptable in the relativistic one, and calls for a revision of the theory. An analogous problem affects Navier-Stokes' theory of viscosity in which the existence of acceleration waves is precluded. To overcome these difficulties various theories had been proposed in the framework of Continuum Mechanics, both in a classical and in a relativistic context.

A different approach to the problem of heat propagation and viscosity is supplied by Kinetic Theory, in which the state of the system is completely defined, in a statistical sense, by a distribution function [**4**], satisfying a transport equation and determining all macroscopic quantities, without any need for assuming any a priori constitutive equation.

In the present work we examine the transport phenomena related to heat propagation and viscosity in the context of Relativistic Kinetic Theory and evaluate the corresponding (finite) speeds of propagation. These problems are also treated in $[\mathbf{3}]_{1,2}$.

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## 2 - Elements of Kinetic Theory

Starting from the distribution function $f$, one defines the 4-flux of particles $N^\mu$ and the energy-momentum tensor $T^{\mu\nu}$ on the basis of the equations [2] (¹)

$$N^\mu(x) = c\int \frac{dp^3}{p^0}\, p^\mu f(x,p)\,, \qquad p^0 = \sqrt{p^2 + m^2c^2}\,, \qquad T^{\mu\nu}(x) = c\int \frac{dp^3}{p^0}\, p^\mu p^\nu f(x,p)\,.$$

Denoting by $U^\mu(x)$ the hydrodynamic 4-velocity field (with normalization $g_{\mu\nu}U^\mu U^\nu = c^2$, $g_{\mu\nu}$ being the metric tensor), let

$$\Delta^{\mu\nu} = g^{\mu\nu} - \frac{1}{c^2}\, U^\mu U^\nu$$

be the spatial projector associated to $U^\mu$. Obviously $U^\mu$ must be related to the dynamical variables describing the gas. Two different characterizations for $U^\mu$ are present in the literature, based on the conditions $\Delta^{\mu\nu}N_\nu = 0$ (Eckart) or $\Delta^{\mu\nu}T_{\nu\sigma}U^\sigma = 0$ (Landau-Lifschitz). Now let us introduce the following quantities

$$n = \frac{1}{c^2}\, N^\mu U_\mu \quad \text{(particle density)}\,, \qquad en = \frac{1}{c^2}\, T^{\mu\nu} U_\mu U_\nu \quad \text{(energy density)}\,,$$

$$I^\mu = (U_\nu T^{\nu\sigma} - hN^\sigma)\Delta^\mu_\sigma \quad (\text{heat flux; } I^\mu U_\mu = 0)\,, \tag{1}$$

where $h = e + p/n$ is the entalpy per particle, and $p$ is the local hydrodynamic pressure, which we assume equal to $nkT$. The space-space projection of the energy-momentum tensor may be split in the following way

$$T^{\sigma\tau}\Delta^\mu_\sigma \Delta^\nu_\tau = -p\Delta^{\mu\nu} + \Pi^{\mu\nu}\,. \tag{2}$$

Accordingly, we have the decomposition

$$T^{\mu\nu} = \overset{(0)}{T}{}^{\mu\nu} + \overset{(1)}{T}{}^{\mu\nu}\,, \qquad \overset{(0)}{T}{}^{\mu\nu} = \frac{1}{c^2}\, en\, U^\mu U^\nu - p\Delta^{\mu\nu}$$

$$\overset{(1)}{T}{}^{\mu\nu} = \frac{1}{c^2}\left((I^\mu + h\Delta^{\mu\sigma}N_\sigma)\, U^\nu + (I^\nu + h\Delta^{\nu\sigma}N_\sigma)\, U^\mu\right) + \Pi^{\mu\nu}\,.$$

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(¹) Greek indices run from 1 to 4; summation over repeated indices is understood; the signature of the metric is $(+,-,-,-)$.

In particular, using Eckart's definition for $U^\mu$, we obtain

$$I^\mu = U_\nu T^{\nu\sigma} \Delta^\mu_\sigma \,, \qquad \overset{(1)}{T}{}^{\mu\nu} = \frac{1}{c^2}(I^\mu U^\nu + I^\nu U^\mu) + \Pi^{\mu\nu}$$

while, using the Landau-Lifschitz one, we obtain

$$I^\mu = -h\Delta^{\mu\nu} N_\nu \,, \qquad \overset{(1)}{T}{}^{\mu\nu} = \Pi^{\mu\nu} \,.$$

For a gas having no interaction with an external field, the distribution function satisfies the Boltzmann equation

$$p^\mu \partial_\mu f(x, p) = C(x, p) \,. \tag{3}$$

Concerning the meaning of the collision term $C$, and more generally, the mathematical problems associated with eq. (3), as well as some approximate methods of solution, the reader is referred to [**1**], [2], [**5**]. Here we report only the solution of the equation (3) based on the so-called fourteen moments approximation. The following equations, in term of the macroscopic quantities previously introduced, hold true in a situation not too far from equilibrium and follow from the Landau-Lifshitz choice of the hydrodynamic 4-velocity

$$Dn + n\nabla_\mu U^\mu - \frac{1}{h}\nabla_\mu I^\mu = 0 \,, \tag{4a}$$

$$\frac{1}{c^2} hnDU^\mu - nk\nabla^\mu T - kT\nabla^\mu n + \nabla_\nu \Pi^{\mu\nu} = 0 \,, \tag{4b}$$

$$\frac{1}{1-\gamma}\frac{DT}{T} - \nabla_\mu U^\mu - \left(\frac{1}{p} - \frac{1}{hn}\right)\nabla_\mu I^\mu = 0, \tag{4c}$$

$$\left(\frac{1}{\eta_v} + \frac{\alpha' D}{nkT}\right)\Pi + \nabla_\nu\left(U^\nu - \frac{\alpha''}{nkT} I^\nu\right) = 0 \,, \tag{4d}$$

$$\left(\frac{1}{T\lambda} + \frac{\beta'' D}{nkTc^2}\right) I^\mu - \left(1 - \frac{kT}{h}\right)\frac{\nabla^\mu T}{T} + \frac{kT}{hn}\nabla^\mu n + \frac{1}{nkT}\left(\gamma''\nabla_\nu \overset{\circ}{\overline{\Pi^{\mu\nu}}} + \alpha''\nabla^\mu \Pi\right) = 0 \,, \tag{4e}$$

$$\left(\frac{1}{2\eta} + \frac{\gamma''' D}{nkT}\right)\overset{\circ}{\overline{\Pi^{\mu\nu}}} - \overset{\circ}{\overline{\nabla^\mu U^\nu}} - \frac{\gamma''}{nkT}\overset{\circ}{\overline{\nabla^\mu I^\nu}} = 0 \tag{4f}$$

in which

$$D = U^\mu \partial_\mu \,, \qquad \nabla^\mu = \Delta^{\mu\varrho}\partial_\varrho \tag{5a, b}$$

and the symbol $\triangleq$ indicates symmetrized traceless spatial projection, while $\eta_v$, $\eta$, $\lambda$ are the transport coefficients, representing bulk and shear viscosity and thermal conductivity respectively; $\alpha'$, $\alpha''$, $\beta''$, $\gamma''$, $\gamma'''$ are functions of the temperature and later we shall give approximate values for them; $\gamma = C_p/C_v$.

## 3 - Propagation of discontinuities

In studying the propagation of acceleration waves, taking (4) into account, we must require continuity of $n$, $U^\mu$, $T$, $\Pi^{\mu\nu}$, $I^\mu$, so that possible discontinuities may only occur in the derivatives of these quantities. Denoting by $\Sigma$ the 3-surface of discontinuity and by $n_\mu$ its normal 1-form, let $\psi_{\alpha_1\ldots\alpha_p}$ be an arbitrary continuous tensor field. Then we have the well known Maxwell relation

$$[\psi_{\alpha_1\ldots\alpha_p,\mu}] = \Omega_{\alpha_1\ldots\alpha_p} n_\mu \,, \tag{6}$$

the left hand side expressing the jump of the derivatives $\psi_{\alpha_1\ldots\alpha_p,\mu}$ on $\Sigma$; $\Omega_{\alpha_1\ldots\alpha_p}$ and $n_\mu$ are defined on $\Sigma$.

In connection with the introduction of $U^\mu$, we may take the spatial resolution of $n_\mu$ in the co-moving frame of reference determined by $U^\mu$ [2]

$$n_\mu = v\frac{U_\mu}{c} + \tilde{n}_\mu \qquad (U^\mu \tilde{n}_\mu = 0)\,. \tag{7}$$

The normalization and orientation of $n_\mu$ is then fixed on the basis of the condition $\tilde{n}^\mu\tilde{n}_\mu = -1$, $v \geqslant 0$. Obviously

$$n^\mu U_\mu = cv\,, \qquad n^\mu n_\mu = v^2 - 1\,. \tag{8}$$

When $v^2 \leqslant 1$ ($n^\alpha n_\alpha \geqslant 0$) the spatial vector $v\tilde{n}^\alpha$ is identified with the 3-velocity of propagation of the discontinuities in the co-moving frame of reference.

From the definitions (5a, b), using (6), (8) we obtain

$$[D\psi_{\alpha_1\ldots\alpha_p}] = U^\mu[\psi_{\alpha_1\ldots\alpha_p,\mu}] = U^\mu\Omega_{\alpha_1\ldots\alpha_p} n_\mu = cv\Omega_{\alpha_1\ldots\alpha_p}\,, \tag{9}$$

$$[\nabla^\mu \psi_{\alpha_1\ldots\alpha_p}] = \Delta^{\mu\nu}[\psi_{\alpha_1\ldots\alpha_p,\nu}] = \Delta^{\mu\nu}\Omega_{\alpha_1\ldots\alpha_p} n_\nu = \Omega_{\alpha_1\ldots\alpha_p}\tilde{n}^\mu\,. \tag{10}$$

We now observe the following facts

(a) $U^\mu U_\mu = c^2 \Rightarrow DU^\mu$ and $[DU^\mu]$ are spatial vectors;

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[2] $v$ is measured in natural units.

(b) assuming that, before the perturbation has come, the gas is in equilibrium, and so $I^\mu = 0$ and $\Pi^{\mu\nu} = 0$, as a consequence of the assumed continuity of $I^\mu$ and $\Pi^{\mu\nu}$, $I^\mu$ and $\Pi^{\mu\nu}$ are equal to zero on $\Sigma$;

(c) $I^\mu$ is spatial (see eq. (1)) and, as a consequence of (b), $[DI^\mu]$ is spatial too;

(d) $\Pi^{\mu\nu}$ is the space-space projection (see eq. (2)) of a symmetric rank-2 tensor and, as a consequence of (b), $[D\Pi^{\mu\nu}]$ is again a space- space tensor.

Now we introduce the following definitions

$$[DU^\mu] = cv\Phi^\mu\,, \qquad [\nabla^\nu U^\mu] = \Phi^\mu \tilde{n}^\nu\,, \tag{11}$$

$$[Dn] = cvN\,, \qquad [\nabla^\mu n] = N\tilde{n}^\mu\,, \tag{12}$$

$$[DI^\mu] = cv\Gamma^\mu\,, \qquad [\nabla^\nu I^\mu] = \Gamma^\mu \tilde{n}^\nu\,, \tag{13}$$

$$[DT] = cv\theta\,, \qquad [\nabla^\mu T] = \theta\tilde{n}^\mu\,, \tag{14}$$

$$[D\Pi^{\mu\nu}] = cvP^{\mu\nu}\,, \qquad [\nabla^\varrho \Pi^{\mu\nu}] = P^{\mu\nu}\tilde{n}^\varrho\,, \tag{15}$$

where all quantities $\Phi^\mu, N, \Gamma^\mu, \theta, P^{\mu\nu}$ are space tensors defined on $\Sigma$.

Taking the discontinuities of both sides of the equations in the system (4) and substituting (11)-(15) into them, we obtain the following algebraic system

$$cvN + n\Phi_\mu \tilde{n}^\mu - \frac{1}{h}\Gamma_\mu \tilde{n}^\mu = 0\,, \qquad \frac{hnv}{c}\Phi^\mu - nk\theta\tilde{n}^\mu - kTN\tilde{n}^\mu + P^{\mu\nu}\tilde{n}_\nu = 0\,, \tag{16a, b}$$

$$\frac{1}{1-\gamma}\frac{cv}{T}\theta - \Phi_\mu\tilde{n}^\mu - \left(\frac{1}{p} - \frac{1}{hn}\right)\Gamma_\mu\tilde{n}^\mu = 0, \tag{16c}$$

$$\frac{\alpha'}{nkT}cvP + \Phi_\mu\tilde{n}^\mu - \frac{\alpha''}{nkT}\Gamma_\mu\tilde{n}^\mu = 0\,, \tag{16d}$$

$$\frac{\beta''}{nkTc}v\Gamma^\mu - \left(1 - \frac{kT}{h}\right)\frac{\theta}{T}\tilde{n}^\mu + \frac{kT}{hn}N\tilde{n}^\mu + \frac{\gamma''}{nkT}(P^{\mu\nu} + P\Delta^{\mu\nu})\tilde{n}_\nu + \frac{\alpha''}{nkT}P\tilde{n}^\mu = 0, \tag{16e}$$

$$\frac{\gamma'''}{nkT}cv(P^{\mu\nu} + P\nabla^{\mu\nu}) - \frac{1}{2}(\Phi^\mu\tilde{n}^\nu + \Phi^\nu\tilde{n}^\mu) \tag{16f}$$

$$+ \frac{1}{3}\Delta^{\mu\nu}\Phi_\alpha\tilde{n}^\alpha - \frac{\gamma''}{nkT}\left(\frac{1}{2}(\Gamma^\mu\tilde{n}^\nu + \Gamma^\nu\tilde{n}^\mu) - \frac{1}{3}\Delta^{\mu\nu}\Gamma_\alpha\tilde{n}^\alpha\right) = 0, \quad \text{where } P = -\frac{1}{3}P^\mu_\mu\,.$$

**4 - Longitudinal waves**

In this case, characterized by the condition $\Phi^\mu = \Phi\tilde{n}^\mu$, ($\Rightarrow \Phi_\mu \tilde{n}^\mu = -\Phi$) eqs. (16) become

$$\text{(17a, b)} \qquad cvN - n\Phi - \frac{1}{h}\Gamma_\mu \tilde{n}^\mu = 0\,, \; \frac{hnv}{c}\Phi\tilde{n}^\mu - nk\theta\tilde{n}^\mu - kTN\tilde{n}^\mu + P^{\mu\nu}\tilde{n}_\nu = 0\,,$$

$$\text{(17c, d)} \qquad \frac{1}{1-\gamma}\frac{cv}{T}\theta + \Phi - \left(\frac{1}{p} - \frac{1}{hn}\right)\Gamma_\mu \tilde{n}^\mu = 0, \; \frac{\alpha'}{nkT}cvP - \Phi - \frac{\alpha''}{nkT}\Gamma_\mu \tilde{n}^\mu = 0,$$

$$\text{(17e)} \; \frac{\beta''}{nkTc}v\Gamma^\mu - \left(1 - \frac{kT}{h}\right)\frac{\theta\tilde{n}^\mu}{T} + \frac{kT}{hn}N\tilde{n}^\mu + \frac{\gamma''}{nkT}(P^{\mu\nu} + P\Delta^{\mu\nu})\tilde{n}_\nu + \frac{\alpha''}{nkT}P\tilde{n}^\mu = 0,$$

$$\text{(17f)} \qquad \frac{\gamma'''}{nkT}cv(P^{\mu\nu} + P\Delta^{\mu\nu}) - \Phi\tilde{n}^\mu\tilde{n}^\nu - \frac{1}{3}\Delta^{\mu\nu}\Phi$$
$$- \frac{\gamma''}{nkT}\left(\frac{1}{2}(\Gamma^\mu\tilde{n}^\nu + \Gamma^\nu\tilde{n}^\mu) - \frac{1}{3}\Delta^{\mu\nu}\Gamma_\alpha\tilde{n}^\alpha\right) = 0\,.$$

Evaluating $P^{\mu\nu}\tilde{n}_\nu$ from (17b) and substituting it in (17e), we deduce that $\Gamma^\mu$ is proportional to $\tilde{n}^\mu$

$$\text{(18)} \qquad \Gamma^\mu = \Gamma\tilde{n}^\mu\,, \qquad \Gamma_\mu\tilde{n}^\mu = -\Gamma\,,$$

corresponding to the longitudinal nature of the thermal discontinuity.

Contracting (17f) with $\tilde{n}_\nu$ we obtain

$$\text{(19)} \qquad P^{\mu\nu}\tilde{n}_\nu = -P\tilde{n}^\mu - \frac{2}{3}\frac{nkT}{\gamma'''cv}\Phi\tilde{n}^\mu - \frac{2}{3}\frac{\gamma''}{\gamma'''cv}\Gamma\tilde{n}^\mu\,.$$

Substituting (18) and (19) into the system (17), we have

$$\text{(20a)} \qquad cvN - n\Phi + \frac{1}{h}\Gamma = 0\,,$$

$$\text{(20b)} \qquad -kTN + \left(\frac{hnv}{c} - \frac{2}{3}\frac{nkT}{\gamma'''cv}\right)\Phi - \frac{2}{3}\frac{\gamma''}{\gamma'''cv}\Gamma - nk\theta - P = 0\,,$$

$$\text{(20c, d)} \qquad \Phi + \left(\frac{1}{p} - \frac{1}{hn}\right)\Gamma + \frac{1}{1-\gamma}\frac{cv}{T}\theta = 0, \qquad -\Phi + \frac{\alpha''}{nkT}\Gamma + \frac{\alpha'}{nkT}cvP = 0\,,$$

$$\text{(20e)} \qquad \frac{kT}{hn}N - \frac{2}{3}\frac{\gamma''}{\gamma'''cv}\Phi + \frac{1}{nkTc}\left(\beta''v - \frac{2}{3}\frac{(\gamma'')^2}{\gamma'''v}\right)\Gamma - \left(1 - \frac{kT}{h}\right)\frac{\theta}{T} + \frac{\alpha''}{nkT}P = 0\,.$$

This is to be regarded as an algebraic system in the 5 unknowns $N$, $\Phi$, $\Gamma$, $\theta$, $P$, all other quantities being functions of them, through the relations

(21a, b) $$\Phi^\mu = \Phi\tilde{n}^\mu\,, \quad \Gamma^\mu = \Gamma\tilde{n}^\mu\,,$$

(21c) $$P^{\mu\nu} = -P\Delta^{\mu\nu} + \frac{nkT}{\gamma''' cv}\left(\tilde{n}^\mu\tilde{n}^\nu + \frac{1}{3}\Delta^{\mu\nu}\right)\left(\Phi + \frac{\gamma''}{nkT}\Gamma\right).$$

The determination of the speeed of propagation is performed by equating to zero the determinant of the matrix of the coefficients in the homogeneous system (20).

In the low temperature range ($z = mc^2/kT \gg 1$), using the approximations [**2**]

(22) $$\alpha' = \frac{6}{5}z^2\,, \quad \beta' = \frac{4}{5}z\,, \quad \gamma = \frac{5}{3}\,, \quad h \simeq mc^2\,,$$
$$\alpha'' = \frac{4}{5}z\,, \quad \beta'' = \frac{2}{5}z\,, \quad \gamma'' = \frac{2}{5}\,, \quad \beta''' = \frac{2}{5}\,, \quad \gamma''' = \frac{1}{2}\,,$$

the speeds of propagation are

$$v_1^2 = 1.96\,\frac{kT}{mc^2}\,, \qquad v_2^2 = 4.12\,\frac{kT}{mc^2}\ (^3)\,.$$

In the high temperature range ($z \ll 1$), using the approximations [**2**]

(23) $$\alpha' = \frac{216}{z^4}\,, \quad \beta' = \frac{6}{z^2}\,, \quad \gamma = \frac{4}{3}\,, \quad h = 4kT\,,$$
$$\alpha'' = \frac{6}{z^2}\,, \quad \beta'' = \frac{5}{4}\,, \quad \gamma'' = \frac{1}{4}\,, \quad \beta''' = \frac{1}{2}\,, \quad \gamma''' = \frac{3}{4}\,,$$

the speeds of propagation are $v_1 = 0.57$, $v_2 = 0.85$.

## 5 - Transverse waves

In this case, characterized by $\Phi_\mu\tilde{n}^\mu = 0$, eqs. (16) become

(24a, b) $$cvN - \frac{1}{h}\Gamma_\mu\,\tilde{n}^\mu = 0\,, \qquad nk\theta + kTN + P^{\mu\nu}\tilde{n}_\mu\tilde{n}_\nu = 0\,,$$

(3) For $T = 10^3\ {}^0K$, $m = 1.6\cdot 10^{-24}$, $k = 1.38\cdot 10^{-16}$, $c = 3\cdot 10^{10}$ (c.g.s. units) we have $cv_1 = 4\cdot 10^5$ cm/sec and $cv_2 = 5.8\cdot 10^5$ cm/sec.

(24c, d) $$\frac{1}{1-\gamma}\,\frac{cv}{T}\,\theta-\Big(\frac{1}{p}-\frac{1}{hn}\Big)\,\Gamma_\mu\tilde{n}^\mu=0\,,\qquad \frac{\alpha'}{nkT}\,cvP-\frac{\alpha''}{nkT}\,\Gamma_\mu\tilde{n}^\mu=0\,,$$

(24e) $$\frac{\beta''}{nkTc}\,v\Gamma_\mu\tilde{n}^\mu+\Big(1-\frac{kT}{h}\Big)\frac{\theta}{T}-\frac{kT}{hn}\,N+\frac{\gamma''}{nkT}\,P^{\mu\nu}\tilde{n}_\mu\tilde{n}_\nu-\frac{\alpha''+\gamma''}{nkT}\,P=0\,,$$

(24f) $$\frac{\gamma'''}{nkT}\,cv(P^{\mu\nu}+P\Delta^{\mu\nu})-\frac{1}{2}\,(\Phi^\mu\tilde{n}^\nu+\Phi^\nu\tilde{n}^\mu)$$

$$+\frac{1}{3}\,\Delta^{\mu\nu}\Phi_\alpha\tilde{n}^\alpha-\frac{\gamma''}{nkT}\Big(\frac{1}{2}\,(\Gamma^\mu\tilde{n}^\nu+\Gamma^\nu\tilde{n}^\mu)-\frac{1}{3}\,\Delta^{\mu\nu}\Gamma_\alpha\tilde{n}^\alpha\Big)=0\,.$$

Expressing $N, \theta, P, P^{\mu\nu}\tilde{n}_\mu\tilde{n}_\nu$ in term of $\Gamma_\mu\tilde{n}^\mu$ through eqs. (24a, b, c, d), and substituting into (24e) we obtain a linear homogeneous expression in $\Gamma_\mu\tilde{n}^\mu$, from which we deduce $\Gamma_\mu\tilde{n}^\mu=0$, so proving the transversal nature of $\Gamma^\mu$. Substituting this result into the system (24), and assuming $v\neq 0$, we have:

(25a, b, c) $$N=0\,,\qquad \frac{hnv}{c}\,\Phi^\mu+P^{\mu\nu}\tilde{n}_\nu=0\,,\qquad \theta=0\,,$$

(25d, e) $$P=0\,,\qquad \frac{\beta''}{c}\,v\Gamma^\mu+\gamma''P^{\mu\nu}\tilde{n}_\nu=0,$$

(25f) $$\frac{\gamma'''}{nkT}\,cvP^{\mu\nu}-\frac{1}{2}\,(\Phi^\mu\tilde{n}^\nu+\Phi^\nu\tilde{n}^\mu)-\frac{1}{2}\,\frac{\gamma''}{nkT}\,(\Gamma^\mu\tilde{n}^\nu+\Gamma^\nu\tilde{n}^\mu)=0\,.$$

The contraction of (25f) with $\tilde{n}_\nu$, in addition with (25b, e), yields an homogeneous linear system in the unknowns $\Phi^\mu, \Gamma^\mu, P^{\mu\nu}\tilde{n}_\nu$, which admits non trivial solutions if and only if the determinant of the coefficients is zero. We have so

$$v^2=\frac{1}{2\gamma'''}\Big(\frac{kT}{h}+\frac{(\gamma'')^2}{\beta''}\Big)\,,\qquad N=0\,,\qquad \theta=0\,,\qquad P=0\,,\qquad \Gamma^\mu=\frac{\gamma''}{\beta''}\,hn\Phi^\mu\,,$$

$$P^{\mu\nu}=\frac{1}{2cv\gamma'''}\,\Big(nkT+\frac{\gamma''}{\beta''}\,hn\Big)\,(\Phi^\mu\tilde{n}^\nu+\Phi^\nu\tilde{n}^\mu)\,.$$

In the low temperature range, using the approximations (22) we obtain

$$v^2=1.4\,\frac{kT}{mc^2}\ (^4)\,.$$

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(4) For $T=10^3\,{}^0K$, $m=1.6\cdot 10^{-24}$, $k=1.38\cdot 10^{-16}$, $c=3\cdot 10^{10}$ (c.g.s. units), we have $cv=3.4\cdot 10^5$ cm/sec.

In the high temperature range, using the approximations (23) we obtain $v^2 = 0.2$.

Concluding, we see that relativistic moments approximation yields a characterization of wave propagation satisfying the relativistic requirement of causality.

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## Riassunto


*Viene studiata la propagazione di onde di accelerazione e termiche in un gas perfetto, usando il metodo dei momenti per la soluzione approssimata dell'equazione di Boltzmann. Vengono esamintati i casi di onde longitudinali e trasversali, a bassa e alta temperatura.*


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