Riv. Mat. Univ. Parma (4) **14** (1988), 249-259

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On the range of $\perp$-decomposable measures (**)

1 - Introduction

The central result in the study of the structure of the range of a measure is Lyapunov's convexity theorem [11]. It was reproved and generalized by several Author's [1], [5], [9] and finds interesting applications in mathematical economics, especially in dealing with the main equilibrium concepts, as the core and the set of Walras allocations (see, e.g., [2], [6]). Let us consider Lyapunov theorem in the following form: the range of a strongly continuous measure is convex and compact. Various issues induce to consider set functions in which additivity assumptions are weakened (recall, for instance, Choquet's capacity theory [4]). Here we focus our attention on the measures which are decomposable with respect to an archimedean t-conorm.

These measures have the important feature that it is possible to define a related integration that results in an actual extension of the Lebesgue theory. (For related results see also [13], [15] and [18].)

It is worth recalling that some set functions, decomposable with respect to associative operations, were used in [7], [8] in order to generalize the concept of information of events. (See also [17].) The operations of t-norm (triangular norm) and t-conorm have their origin from generalized triangle inequalities, due to Menger [12]. (For further developments see, e.g., [14].)

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(**) Ricevuto: 10-X-1988.

Our present concern is to state some properties of the range of the measures which are decomposable with respect to an archimedean t-conorm. In particular we shall prove that Lyapunov's theorem in the form above can be restated.

2 - Prerequisites

Let us recall some definitions and results, following [**14**], [**18**] and references therein. For the other basic notions of measure theory see [3].

A binary operation $\perp$ on the real unit interval $J=[0,\ 1]$ is said to be a *t-conorm* if $\perp$ is non decreasing in each argument, commutative, associative and has 0 as unit.

A t-conorm $\perp$ is said to be *archimedean* if it is continuous and such that $\perp(x,\ x)>x$, for every $x\in(0,\ 1)$. Therefore the structure $(J,\ \perp)$ is an archimedean abelian semigroup with unit 0.

An archimedean t-conorm $\perp$ is called *strict* if it is strictly increasing in $(0,\ 1)\times(0,\ 1)$.

The following Ling's representation theorem holds.

Theorem 2.1 [**10**]. A binary operation $\perp$ on J is an archimedean t-conorm if and only if there exists an increasing and continuous function $g:J\to[0,\ \infty]$, with $g(0)=0$, such that

$$\perp(x,\ y)=g^{(-1)}(g(x)+g(y))$$

where $g^{(-1)}$ is the pseudo-inverse of g defined by

$$g^{(-1)}(x)=g^{-1}(\min(x,\ g(1)))\,.$$

Moreover $\perp$ is strict if and only if $g(1)=\infty$.

The function g, called an *additive generator* of $\perp$, is unique up to a positive constant factor.

Let $(X,\ \underline{A})$ be a measurable space. A set function $\mu:\underline{A}\to J$, with $\mu(\emptyset)=0$ and $\mu(X)=1$, is called [**18**]

(a) decomposable measure with respect to a t-conorm $\perp$, or *$\perp$-decomposable* *measure*, if

$$\mu(a \dot{\cup} B) = \mu(A) \perp \mu(B).$$

(b) σ-$\perp$-*decomposable measure*, if

$$\mu(\dot{\bigcup_{n=1}^{\infty}} A_n) = \mathop{\perp}_{n=1}^{\infty} \mu(A_n).$$

The notation $(\sigma\text{-})\perp$-decomposable will stand for $\perp$-or σ-$\perp$-decomposable.

Proposition 2.2 [18]. (a) If μ is $\perp$-decomposable, then μ is monotone. (b) μ is $\perp$-decomposable if and only if

$$\mu(A \cup B) \perp \mu(A \cap B) = \mu(A) \perp \mu(B)$$

for every A, $B \in \underline{A}$.

For any t-conorm $\perp$, the following operations are defined

$$b \dot{-} a = \inf\{y / a \perp y \geqslant b\} \qquad a' = 1 \dot{-} a.$$

The following classification is valid for $(\sigma\text{-})$ $\perp$-decomposable measures with respect to archimedean t-conorms [18].

(S): $\perp$ strict. Then $g \circ \mu : \underline{A} \to [0, \infty]$ is an infinite $(\sigma\text{-})$ additive measure, whenever μ is a $(\sigma\text{-})$ $\perp$-decomposable one.

(NSA): $\perp$ non-strict archimedean and $g \circ \mu : \underline{A} \to [0, g(1)]$ a finite $(\sigma\text{-})$ additive measure with $(g \circ \mu)(X) = g(1)$.

(NSP): $\perp$ non-strict archimedean and $g \circ \mu$ a finite measure with $(g \circ \mu)(X) = g(1)$, which is only pseudo $(\sigma\text{-})$ additive, i.e., it is possible that

$$(g \circ \mu)(\dot{\bigcup_k} A_k) = g(1) < \sum_k (g \circ \mu) A_k.$$

Let $\perp$ be an archimedean t-conorm and μ a $\perp$-decomposable measure. Then [18]:

P1. $A \subseteq B$ implies $\mu(B-A) \geqslant \mu(B) \dot{-} \mu(A)$. Under the additional conditions for (S) with $\mu(A)<1$ or (NSP) with $\mu(B)<1$, resp., it is $\mu(B-A)=\mu(B) \dot{-} \mu(A)$.

Let us mention finally the following properties.

P2. If μ is a $\perp$-decomposable measure, for every $A, B \in \underline{A}$ it is $\mu(A \cup B) \leqslant \mu(A) \perp \mu(B)$.

If $\{A_n\}$ is a sequence of measurable subsets of X, with $A_i \cap A_j = \emptyset$ $(i \neq j)$ for every $A \in \underline{A}$ such that $A \supseteq \bigcup_{n=1}^{\infty} A_n$, it is

$$\mathop{\perp}_{n=1}^{\infty} \mu(A_n) \leqslant \mu(A).$$

In particular $\mathop{\perp}_{n=1}^{\infty} \mu(A_n) \leqslant \mu(\bigcup_{n=1}^{\infty} A_n)$. If μ is σ-$\perp$-decomposable and $\{A_n\}$ is an arbitrary sequence of measurable subsets of $\underline{A}$, then

$$\mu(\bigcup_{n=1}^{\infty} A_n) \leqslant \mathop{\perp}_{n=1}^{\infty} \mu(A_n).$$

P3. For any non-strict archimedean t-conorm with additive generator g we have

$$b \dot{-} a = g^{-1}(g(b)-g(a)) \qquad \text{for } a \leqslant b.$$

For any strict archimedean t-conorm $\perp$ with additive generator g it is

$$b \dot{-} a = \begin{cases} g^{-1}(g(b)-g(a)) & \text{if } a \leqslant b \text{ and } a<1 \\ 0 & \text{otherwise} \end{cases}$$

3 - Finite and infinite ranges

Let μ be a $\perp$-decomposable measure on a measurable space $(X, \underline{A})$. Under suitable hypotheses on μ it is possible to infer that $\underline{A}$ is necessarily finite, and therefore the range $R(\mu)$ of μ is finite.

Lemma 3.1. *Let $(X, \underline{A})$ be a measurable space, in which a decomposable measure μ with respect to an archimedean non strict t-conorm $\perp$ is defined. If there exists a constant d such that*

(1) $$0 < \mu(A) < d \qquad d \in (0, 1)$$

for every $A \in \underline{A} - \{X, \emptyset\}$, then the σ-algebra $\underline{A}$ is finite.

Proof. Let us start by proving that if $\{A_j\}_{j \in F}$ is a disjoint family of elements in $\underline{A}$, then the index set F is finite. From (1) it follows the existence of $c \in (0, 1)$, such that

$$\mu(A) > c \qquad \forall A \in \underline{A} - \{\emptyset\};$$

indeed, by proposition P3,

$$\mu(A) = \mu(X - (X - A)) \geqslant 1 \dot{-} \mu(X - A) \geqslant 1 \dot{-} d > 0 .$$

Let us put

$$F_n = \{i \in F : \frac{1}{n+1} < \mu(A_i) \leqslant \frac{1}{n}\} .$$

The set F_n is finite for every n. Indeed if there is $\{i_1 \ldots, i_m\} \subseteq F_n$ such that $\bigcup_{J=1}^{m} A_{i_j} = X$, then $\operatorname{card} F = \operatorname{card} F_n = m$. Otherwise let us suppose that for every m-tuple in F_n it is $\bigcup_{J=1}^{m} A_{i_j} \neq X$. Put

$$\left(\frac{1}{n+1}\right)^{(k)} = \frac{1}{n+1} \perp \underset{(k \text{ times})}{\cdots} \perp \frac{1}{n+1} .$$

It is $\lim_k \left(\frac{1}{n+1}\right)^{(k)} = 1$, and so there is k_n such that

(2) $$\left(\frac{1}{n+1}\right)^{(k_n)} > d$$

(k_n will denote the minimum integer satisfying (2)). It is easy to check that

(3) $$\operatorname{card} F_n \leqslant k_n .$$

Indeed $\operatorname{card} F_n \geqslant k_n + 1$ implies

$$(\bigcup_{i \in F'_n} A_i) = \mathop{\perp}_{i \in F'_n} \mu(A_i) \geqslant (\frac{1}{n+1})^{(k_n+1)} > d$$

for some $F'_n \subseteq F_n$, $\operatorname{card} F'_n = k_n + 1$, that contradicts (1). Let r be the integer $1/(r+1) \leqslant c \leqslant 1/r$, then

$$F = \bigcup_{i=1}^{r} F_i \qquad \operatorname{card} F \leqslant k_1 + k_2 + \ldots + k_r .$$

Let now put

$$D = \{(A_1, \ldots, A_q) : A_i \in \underline{A} \text{ and } A_i \cap A_j = \emptyset\}$$

and show that there exists a maximal family $(A_1, \ldots, A_{\bar{q}})$ in D, i.e. if $(B_1, \ldots, B_q) \in D$, then $q \leqslant \bar{q}$.

As in the discussion above, the following cases occur for $(A_1, \ldots, A_q) \in D$:

1. For every n and $(j_1, \ldots, j_m) \subseteq F_n$, it is

$$\bigcup_{p=1}^{m} A_{j_p} \neq X$$

and therefore $q \leqslant k_1 + \ldots + k_r$, where r and the k_i's are independent of the particular family $(A_1, \ldots, A_q)$.

2. There are $\bar{n} \in \{1, \ldots, r\}$ and $(j_1, \ldots, j_m) \subseteq F_{\bar{n}}$ such that

$$\bigcup_{p=1}^{m} A_{j_p} = X .$$

Then $A_1, \ldots, A_q$ coincide with $A_{j_1}, \ldots, A_{j_m}$ with $q = m$. Let now show $q \leqslant k_{\bar{n}} + 1$. Indeed $(A_{j_1}, \ldots, A_{j_{m-1}})$ consists of disjoint elements and furthermore $\bigcup_{p=1}^{m-1} A_{j_p} \neq X$; by (3) $q - 1 = m - 1 \leqslant k_{\bar{n}}$; i.e. $q \leqslant k_{\bar{n}} + 1$.

Also in this case the upper bound for q is independent of the particular family in D.

Let observe that a maximal family in D is a partition of X and furthermore it is unique (if $(A_1, \ldots, A_{\bar{q}})$ and $(B_1, \ldots, B_{\bar{q}})$ were two distinct maximal families, then the family of their intersections should be in D). Finally $\underline{A}$ coincides with the σ-algebra generated by $(A_1, \ldots, A_{\bar{q}})$.

Thus the range $R(\mu)$ of the measure μ is finite, as it is

$$R(\mu) = \{0\} \cup \{ \mathop{\perp}_{i=1}^{p} \mu(A_{j_i}),\ \{j_1, \ldots, j_p\} \subseteq \{1, \ldots, \bar{q}\}\}.$$

The following is a topological property of the range of some ⊥-decomposable measures.

Theorem 3.2. *Let μ be a ⊥-decomposable measure on $(X, \underline{A})$, with respect to an archimedean non strict t-conorm ⊥, such that $0 < \mu(A) < 1$, for every $A \in \underline{A} - \{\emptyset, X\}$. If $R(\mu)$ is infinite, then 1 is an accumulation point of $R(\mu)$. Under the additional condition* (NSA), *$R(\mu)$ is dense in itself.*

Proof. *Observe first that 1 is an accumulation point of $R(\mu)$. Indeed if 1 were not an accumulation point of $R(\mu)$, then, for some $d \in (0, 1)$, $0 < \mu(A) < d$ for every $A \in \underline{A} - \{\emptyset, X\}$ and $R(\mu)$ would be finite by the previous Lemma. Then there exists a sequence $\{A_n\}$ in $\underline{A}$, with $0 < \mu(A_n) < 1$, and $\lim_n \mu(A_n) = 1$. If condition (NSA) is satisfied, by property P1 and continuity of $\dot{-}$,*

$$\lim_n \mu(X - A_n) = \lim_n (\mu(X) \dot{-} \mu(A_n)) = 0.$$

Furthermore $\mu(X) \dot{-} \mu(A_n) = g^{-1}(g(1) - g(\mu(A_n))) > 0$ and 0 is an accumulation point of $R(\mu)$ too.

Assign now the value $\mu(A)$, $0 < \mu(A) < 1$; as $R(\mu)$ is infinite μ will take infinite values either on A, or $X - A$. Let μ take infinite values on A. Then the set function $\mu_A : B \in \underline{A},\ B \subseteq A \to \mu(B)$ has an infinite range and $0 < \mu_A(B) < \mu(A)$, for every $B \in \underline{A} - \{\emptyset, A\}$.

It does not occur $0 < \mu_A(B) < d < \mu(A)$, for some d and for every $B \subseteq A$, $B \in \underline{A} - \{\emptyset, A\}$, because the σ-algebra $\underline{A}_A$, formed by the measurable subsets of A, contains infinite elements. Therefore $\mu(A)$ is an accumulation point of $R(\mu_A)$ and $R(\mu)$.

If μ takes infinite values on the complementary of A, applying the same line of the argument above to the restriction μ_{X-A} of μ to $(X - A) \cap \underline{A}$, one can prove that 0 is an accumulation point of the range of μ_{X-A}. Then there exists a sequence $\{B_n\}$ in $(X-A)\cap \underline{A}$, with $\lim_n \mu(B_n)=0$ and $\mu(B_n)>0$. We have

$$\lim_n \mu(A\cup B_n)=\lim_n (\mu(A)\perp \mu(B_n))=\mu(A).$$

Furthermore, as $\mu(B_n)\to 0$ and $\mu(A)<1$,

$$\mu(A\cup B_n)=g^{(-1)}(g(\mu(A))+g(\mu(B_n)))>g^{-1}(g(\mu(A)))=\mu(A).$$

Thus $\mu(A)$ is an accumulation point of $R(\mu)$.

Example. Let us exhibit a measure fulfilling the hypotheses of the previous theorem. First define the binary operation on J

$$\perp\colon (a,\ b)\in J^2 \to \min(1/\log_{1/2}((\frac{1}{2})^{1/a}+(\frac{1}{2})^{1/b}),\ 1)$$

with the assumptions $1/0=+\infty$, $(1/2)^{+\infty}=0$, $\log_{1/2}(0)=+\infty$.

The operation $\perp$ is a non-strict archimedean t-conorm with additive generator:

$$g\colon x\in J = \begin{cases} 0 & \text{if } x=0 \\ (1/2)^{1/x} & \text{if } x>0 \end{cases}$$

whose pseudo-inverse is

$$g^{(-1)}\colon y\in[0,\ +\infty] = \begin{cases} 0 & \text{if } y=0 \\ 1/\log_{1/2} y & \text{if } 0<y\leqslant 1/2=g(1) \\ 1 & \text{if } y>1/2. \end{cases}$$

Let us consider the set $S=\{1/n\}_{\substack{n\in N\\ n>1}}$ and the measurable space $(S,\ \mathscr{P}(S))$. Define the set function

$$\mu\colon A\in \mathscr{P}(S) = \begin{cases} 0 & \text{if } A=\emptyset \\ x_0 & \text{if } A=\{x_0\} \\ \perp_{x\in A}\, x & \text{otherwise.} \end{cases}$$

One can check that $\mu(S)=1$, $0<\mu(A)<1$ for $\emptyset\neq A\neq S$ and μ is σ-$\perp$-

decomposable of type (NSA). Indeed: if $\{A_k\}$ is a sequence in $\mathscr{P}(S)$, with $A_k \cap A_h = \emptyset$ and $\cup A_k = S$,

$$\sum_{k=1}^{\infty} (g(\mu(A_k)) = \sum_{k=1}^{\infty} g(\underset{r_{m_j}^{(k)} \in A_k}{\perp} r_{m_j}^{(k)}) = \sum_{k=1}^{\infty} g(g^{-1}(\sum_{r_{m_j}^{(k)} \in A_k} g(r_{m_j}^{(k)})) \tag{4}$$

$$= \sum_{k=1}^{\infty} \sum_{r_{m_j}^{(k)} \in A_k} g(r_{m_j}^{(k)}) = \sum_{n=1}^{\infty} g(r_n) = g(1)$$

because $g(r_{m_j}^k) < g(1)$ and (4) is with positive terms. On the other hand

$$g(\mu(\bigcup_{k=1}^{\infty} A_k)) = g(\mu(S)) = g(1)\,.$$

Then the additivity of $g \circ \mu$. By Theorem 3.2, $\overline{R(\mu)} = J$.

4 - A Lyapunov-like theorem for ⊥-decomposable measures

The definition of strong continuity for a ⊥-decomposable measure is the same for additive measures [3]. A ⊥-decomposable measure μ is *strongly continuous* on $\underline{A}$ if for every $\varepsilon > 0$, there is a finite partition $P = \{A_1, \ldots, A_n\}$, $A_i \in \underline{A}$, of X, such that $\mu(A_i) < \varepsilon$ $(i = 1, \ldots, n)$.

Let us state a 1-dimensional version of Lyapunov's theorem for ⊥-decomposable measures

Theorem 4.1. *Let $(A, \underline{A})$ be a measurable space and μ a ⊥-decomposable measure with respect to the archimedean t-conorm ⊥. If μ is strongly continuous, then $R(\mu) = [0, 1]$.*

Proof. The proof develops analogously to the additive case (see, e.g. [3]). Let $a \in (0, 1)$. By the strong continuity of μ there is a finite partition P of X into measurable sets such that the measure of every element in P is less than 1/2. Let A_1 denote the largest union of elements in P such that $\mu(A_1) \leqslant a$. Evidently $A_1 \neq X$.

If $\mu(A_1) = a$, then the theorem follows. If $\mu(A_1) < a$, let B_1 be an element of P disjoint from A_1. Of course $0 < \mu(B_1)$ and $\mu(A_1 \cup B_1) > a$. Since the restriction of μ to $B_1 \cap \underline{A}$ is strongly continuous, there exists a finite partition Q of B_1, having all elements with measure less than $1/(2^2)$. Let A_2 denote the largest union of elements in Q such that $\mu(A_2) \leqslant a \dot{-} \mu(A_1)$. Observe that $a \dot{-} \mu(A_1) > 0$, by P3. If $\mu(A_2) = a \dot{-} \mu(A_1)$, then $\mu(A_1 \cup A_2) = a$, by P3, and the theorem follows. Let $\mu(A_2) < a \dot{-} \mu(A_1)$. Observe first that $a \dot{-} \mu(A_1) < \mu(B_1)$ and therefore $A_2 \neq B_1$. Let B_2 be an element of Q which is disjoint from A_2. It is $0 < \mu(B_2) < 1/(2^2)$ and $\mu(A_2 \cup B_2) > a \dot{-} \mu(A_1)$. Therefore $\mu(A_1) \perp \mu(A_2) \perp \mu(B_2) > a$. The procedure can be iterated and the sets A_3, B_3, A_4, B_4, ... constructed. One of the following cases occurs: either $\mu(\bigcup_{i=1}^{k} A_i) = a$, for some k, or two infinite sequences $\{A_n\}$ and $\{B_n\}$ are determined such that

(a) $A_n \cap B_n + \emptyset$ (b) $A_{n+1} \cup B_{n+1} \subsetneq B_n$

(c) $\mu(A_1) \perp \ldots \perp \mu(A_n) < a$ (d) $\mu(A_1) \perp \ldots \perp \mu(A_n) \perp \mu(B_n) > a$

(e) $0 < \mu(B_n) < 1/(2^n)$.

Let us show that $\perp_{n=1}^{\infty} \mu(A_n) = a$. Indeed, by (c), $\perp_{n=1}^{\infty} \mu(A_n) \leqslant a$. The reverse inequality is implied by P1 and (d)

$$\lim_k \perp_{n=1}^{k} \mu(A_n) \geqslant \lim_k \left(\perp_{n=1}^{k} \mu(A_n) \perp \mu(B_k) \right) \dot{-} \mu(B_k) \geqslant a \dot{-} \lim_k \mu(B_k) = a .$$

Let us observe that if measure μ is σ-$\perp$-decomposable, $\mu(\bigcup_{n=1}^{\infty} A_n) = \perp_{n=1}^{\infty} \mu(A_n)$ and, obviously, $\bigcup_{n=1}^{\infty} A_n$ is a measurable set on which μ takes value a. Also in the general case, when μ is $\perp$-decomposable; it happens that

$$\mu(\bigcup_{n=1}^{\infty} A_n) = a . \tag{5}$$

Indeed

$$\mu(\bigcup_{n=1}^{\infty} A_n) \leqslant \perp_{n=1}^{k+1} \mu(A_n) \perp \mu(B_{k+1}) \leqslant \perp_{n=1}^{k+1} \mu(A_n) \perp \frac{1}{2^{k+1}} .$$

Finally, for $k \to \infty$, $\mu(\bigcup_{n=1}^{\infty} A_n) \leqslant \perp_{n=1}^{\infty} \mu(A_n) = a$ and, recalling P2, equality (5) follows.

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Abstract

Topological properties of the range of measures which are decomposable with respect to an archimedean t-conorm are studied. A Lyapunov's theorem is proved for such a class of measures.

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