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Curvature characterizations in contact geometry (**)

1 - Introduction

Let (M, g) be a C^∞ Riemannian manifold and R its Riemann curvature tensor. It is an interesting problem to determine what kind of Riemannian manifolds may be determined by special pointwise expressions for R. The following example is classical: Let (M, g) be a connected Riemannian manifold with dimension >2 and suppose that its curvature tensor has the following pointwise expression

$$R_{XY}Z = \lambda\{g(X, Z)Y - g(Y, Z)X\}$$

where λ is a C^∞ function on M. Then (M, g) is a space of constant sectional curvature. Another typical example is given in $[5]_2$: Let (M, g) be a connected Riemannian manifold with $\dim M > 2$ whose curvature tensor at each point is that of an irreducible symmetric space $(N, \mathring{g})$. Then (M, g) is locally symmetric and locally isometric to that model space $(N, \mathring{g})$. (We refer to $[5]_3$ for extensions of this result.)

Further, when the manifold (M, g) is equipped with some additional structure, then it is sometimes possible to derive conclusions for this structure too from the special form of R. In $[5]_1$ an example is given for an almost Hermitian manifold and a quaternionic analog is proved in [4]. In [2] we treated a similar problem in contact geometry. The main purpose of this paper is to extend

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this result and to derive other algebraic characterizations for some classes of almost contact metric structures and manifolds by means of special expressions for the curvature tensor.

We start with some preliminaires in 2 and prove the main result in 3. We refer to [1], [3], [6] for more details and examples.

2 - Preliminaries

Let M be a $(2n+1)$-dimensional C^∞ manifold and $\mathcal{X}(M)$ the Lie algebra of C^∞ vector fields on M. An *almost contact structure* on M is defined by a C^∞ (1, 1)-tensor field φ, a C^∞ vector field ξ and a C^∞ one-form η on M such that for any point $m \in M$ we have

$$\varphi_m^2 = -I + \eta_m \otimes \xi_m \qquad \eta_m(\xi_m) = 1$$

where I denotes the identity transformation of the tangent space $T_m M$ of M at m. This implies $\varphi\xi = 0$, $\eta \circ \varphi = 0$. Manifolds equipped with an almost contact structure are called *almost contact manifolds*. A Riemannian manifold M with metric tensor g and an almost contact structure (φ, ξ, η) such that

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y)$$

for all $X, Y \in \mathcal{X}(M)$, is called an *almost contact metric manifold*. Note that the existence of an almost contact structure on M is equivalent with the existence of a reduction of the structural group of the tangent bundle to $U(n) \times 1$.

The Sasaki form ϕ of an almost contact metric manifold $(M, g, \varphi, \xi, \eta)$ is defined by

$$\phi(X, Y) = g(X, \varphi Y) \quad \text{for all } X, Y \in \mathcal{X}(M).$$

The associated structure is cosymplectic if $d\phi = d\eta = 0$ and then $(M, g, \varphi, \xi, \eta)$ is called an *almost co-Kählerian manifold*. When $\phi = d\eta$ the structure is a contact structure and $(M, g, \varphi, \xi, \eta)$ an *almost Sasakian manifold*. The almost contact metric manifold is said to be *almost α-Sasakian* if $\phi = \frac{1}{\alpha} d\eta$, $\alpha \in \mathbb{R}_0$.

An almost contact structure is said to be *normal* if

$$[\varphi, \varphi] + 2d\eta \otimes \xi = 0$$

where $[\varphi, \varphi]$ denotes the Nijenhuis tensor of φ. A *co-Kählerian manifold* is a normal almost co-Kähler manifold and an *α-Sasakian manifold* is a normal almost α-Sasakian manifold. Here we have the following useful characterizations [3]

Proposition 1. *Let $(M, g, \varphi, \xi, \eta)$ be an almost contact metric manifold with Riemannian connection ∇. Then*

(i) *M is co-Kählerian if and only if $\nabla\varphi = 0$;*

(ii) *M is α-Sasakian if and only if for all $X, Y \in \mathfrak{X}(M)$*

$$(\nabla_X \varphi) Y = \alpha\{g(X, Y)\xi - \eta(Y)X\}. \tag{1}$$

Note that ξ *is a Killing vector field on co-Kählerian and α-Sasakian manifolds.*

Further, let R denote the Riemannian curvature tensor determined by

$$R_{XY}Z = \nabla_{[X,Y]}Z - [\nabla_X, \nabla_Y]Z$$

for $X, Y, Z \in \mathfrak{X}(M)$. An almost contact metric manifold is said to be an *almost $C(\alpha)$-manifold* if there exists an $\alpha \in \mathbf{R}$ such that for all $X, Y, Z, W \in \mathfrak{X}(M)$ we have

$$\begin{aligned} R_{XYZW} = R_{XY\varphi Z\varphi W} + \alpha\{&g(X, Z)g(Y, W) - g(X, W)g(Y, Z) \\ &- g(X, \varphi Z)g(Y, \varphi W) + g(X, \varphi W)g(Y, \varphi Z)\} \end{aligned} \tag{2}$$

where $R_{XYZW} = g(R_{XY}Z, W)$. A normal almost $C(\alpha)$-manifold is a *$C(\alpha)$-manifold.* Then we have [3]

Proposition 2. *An α-Sasakian manifold is a $C(\alpha^2)$-manifold and a co-Kählerian manifold is a $C(0)$-manifold.*

A plane section in $T_m M^{2n+1}$, $m \in M$, is called a *φ-section* if it possesses an orthonormal basis of the form $\{X, \varphi X\}$, where $X \in T_m M^{2n+1}$ is vector orthogonal to ξ_m. The sectional curvature

$$K(X, \varphi X) = H(X) = R(X, \varphi X, X, \varphi X)$$

is called the associated *φ-sectional curvature*. For a $C(\alpha)$-manifold, the φ-sectional curvature completely determines the curvature and moreover, for $\dim M \geqslant 5$, if the φ-sectional curvature at any point of a $C(\alpha)$-manifold is independent of the choice of φ-section at the point, then it is constant on the connected manifold. A $C(\alpha)$-manifold of constant φ-sectional curvature is called a *$C(\alpha)$-space form*. Its curvature tensor is given by

$$(3)\qquad R_{XY}Z = \frac{\lambda + 3\alpha^2}{4}\{g(X, Z)\,Y - g(Y, Z)\,W\} + \frac{\lambda - \alpha^2}{4}\{\eta(Y)\,\eta(Z)\,X - \eta(X)\,\eta(Z)\,Y$$

$$- g(X, Z)\,\eta(Y)\,\xi + g(Y, Z)\,\eta(X)\,\xi - g(Z, \varphi Y)\,\varphi X + g(Z, \varphi X)\,\varphi Y - 2g(X, \varphi Y)\,\varphi Z\}$$

where λ is the constant φ-sectional curvature.

3 - The main result

Our purpose is to start with an almost contact metric manifold whose Riemannian curvature tensor has, at each point, the form (3) and to derive a result for the almost contact metric structure and for the manifold itself. More specifically, we prove the following

Theorem. *Let $(M, g, \varphi, \xi, \eta)$ be a connected almost contact metric manifold such that $g(X, \nabla_X \xi) = 0$ for any vector field X orthogonal to ξ and with* $\dim M = 2n + 1 \geqslant 5$. *Suppose further that the Riemann curvature tensor R has the form*

$$(4)\qquad R = f\varphi_1 + h\varphi_2$$

where f and h are C^∞ functions on M such that h is not identically zero and where

$$(5)\qquad \varphi_1(X, Y)\,Z = g(X, Z)\,Y - g(Y, Z)\,X$$

$$(6)\qquad \varphi_2(X, Y)\,Z = \eta(Y)\,\eta(Z)\,X - \eta(X)\,\eta(Z)\,Y - g(X, Z)\,\eta(Y)\,\xi$$

$$+ g(Y, Z)\,\eta(X)\,\xi - g(Z, \varphi Y)\,\varphi X + g(Z, \varphi X)\,\varphi Y - 2g(X, \varphi Y)\,\varphi Z\,.$$

Then f and h are constant and $f-h$ is nonnegative. Moreover, if $f-h=0$, (g, φ, ξ, η) is a co-Kählerian structure and $(M, g, \varphi, \xi, \eta)$ a co-Kähler space form. Further, if $f-h=\alpha^2>0$, then (g, φ, ξ, η) or $(g, -\varphi, \xi, \eta)$ is an α-Sasakian structure and the corresponding manifold is a $C(\alpha^2)$-space form.

The proof will be given in several steps. The first three use the second Bianchi identity

$$\underset{V,X,Y}{\mathfrak{S}}(\nabla_V R)_{XYZW}=0.$$

Proof. *Step* 1. Let X, Y, Z, V, W be vector fields orthogonal to ξ. Using (4), (5), (6), a straightforward computation leads to

$$\begin{aligned}(7)\qquad (\nabla_V R)_{XYZW} &= V(f)\{g(X, Z)g(Y, W)-g(Y, Z)g(X, W)\}\\ &+V(h)\{-g(Z, \varphi Y)g(\varphi X, W)+g(Z, \varphi X)g(\varphi Y, W)-2g(X, \varphi Y)g(\varphi Z, W)\}\\ &+h\{-g(Z, (\nabla_V\varphi)Y)g(\varphi X, W)-g(Z, \varphi Y)g((\nabla_V\varphi)X, W)\\ &+g(Z, (\nabla_V\varphi)X)g(\varphi Y, W)+g(Z, \varphi X)g((\nabla_V\varphi)Y, W)\\ &-2g(X, (\nabla_V\varphi)Y)g(\varphi Z, W)-2g(X, \varphi Y)g((\nabla_V\varphi)Z, W)\}.\end{aligned}$$

Since $\dim M\geqslant 5$, we can choose Y orthogonal to ξ, X, φX. Further, we put $\|X\|=\|Y\|=1$ and $V=\varphi Y$, $Z=X$. Then, the second Bianchi identity and (7) yield

$$\begin{aligned}(8)\qquad &(\varphi Y)(f)g(Y, W)-Y(f)g(\varphi Y, W)+2X(h)g(\varphi X, W)\\ &+h\{-3g((\nabla_{\varphi Y}\varphi)Y, X)g(\varphi X, W)+g((\nabla_{\varphi Y}\varphi)X, X)g(\varphi Y, W)\\ &-g((\nabla_X\varphi)\varphi Y, X)g(\varphi Y, W)-g((\nabla_X\varphi)Y, X)g(Y, W)\\ &-2g((\nabla_X\varphi)\varphi Y, Y)g(\varphi X, W)+2g((\nabla_X\varphi)X, W)+g((\nabla_Y\varphi)X, X)g(Y, W)\\ &+g((\nabla_Y\varphi)\varphi Y, X)g(\varphi X, W)-2g((\nabla_Y\varphi)X, \varphi Y)g(\varphi X, W)\}=0.\end{aligned}$$

Now, we put successively $W=\varphi X$, $W=Y$ and $W=\varphi Y$ in (8). This gives

$$(9)\qquad \begin{aligned} &2X(h)-3h\{g((\nabla_{\varphi Y}\varphi)Y, X)+g((\nabla_Y\varphi)X, \varphi Y)=0\\ &Y(f)-3hg((\nabla_X\varphi)X, \varphi Y)=0 \end{aligned}$$

for any X *orthogonal to* ξ, Y *orthogonal to* ξ, X, φX and $\|X\|=\|Y\|=1$. By interchanging the role of X and Y in (9) we get from (9)

$$(10)\qquad X(f+h)=0.$$

Step 2. Let X, Y, Z, W be orthogonal to ξ, $V=\xi$ and proceed as in Step 1. This gives

$$(11)\qquad (\nabla_\xi R)_{XYZW}=\xi(f)\{g(X, Z)g(Y, W)-g(Y, Z)g(X, W)\}$$

$$+\xi(h)\{-g(Z, \varphi Y)g(\varphi X, W)+g(Z, \varphi X)g(\varphi Y, W)-2g(X, \varphi Y)g(\varphi Z, W)\}$$

$$+h\{-g((\nabla_\xi\varphi)Y, Z)g(\varphi X, W)-g(Z, \varphi Y)g((\nabla_\xi\varphi)X, W)$$

$$+g((\nabla_\xi\varphi)X, Z)g(\varphi Y, W)+g(Z, \varphi X)g((\nabla_\xi\varphi)Y, W)$$

$$-2g((\nabla_\xi\varphi)Y, X)g(\varphi Z, W)-2g(X, \varphi Y)g((\nabla_\xi\varphi)Z, W)$$

$$-g(Y, Z)g(\nabla_X\xi, W)+g(Y, W)g(\nabla_X\xi, Z)+g(\varphi\nabla_X\xi, Z)g(\varphi Y, W)$$

$$-g(Z, \varphi Y)g(\varphi\nabla_X\xi, W)+2g(\varphi\nabla_X\xi, Y)g(\varphi Z, W)+g(X, Z)g(\nabla_Y\xi, W)$$

$$-g(X, W)g(\nabla_Y\xi, Z)-g(\varphi\nabla_Y\xi, Z)g(\varphi X, W)$$

$$+g(Z, \varphi X)g(\varphi\nabla_Y\xi, W)-2g(\varphi\nabla_Y\xi, X)g(\varphi Z, W)\}.$$

As before we put $\|X\|=\|Y\|=1$ and take Y orthogonal to ξ, X, φX. Then, we take first $Z=X$ in (11). This yields for $W=\varphi X$, $W=Y$ and $W=\varphi Y$:

$$(12)\qquad \begin{aligned} &\xi(f)+h\{g(X, \nabla_X\xi)+g(Y, \nabla_Y\xi)\}=0 \qquad h\{g(\nabla_X\xi, \varphi X)-g(\nabla_Y\xi, \varphi Y)\}=0\\ &h\{3g((\nabla_\xi\varphi)Y, X)+2g(\nabla_X\xi, \varphi Y)-4g(\nabla_Y\xi, \varphi X)\}=0. \end{aligned}$$

Proceeding in the same way for $Z=\varphi X$ and $W=X$, $W=Y$ and $W=\varphi Y$, we obtain

$$
(13)\quad \begin{aligned}
&\xi(h)+h\{g(\nabla_X\xi,\ X)+g(\nabla_Y\xi,\ Y)\}=0 \qquad h\{g(\nabla_X\xi,\ \varphi X)-g(\nabla_Y\xi,\ \varphi Y)\}=0\\
&h\{3g((\nabla_\xi\varphi)Y,\ X)+2g(\nabla_X\xi,\ \varphi Y)-4g(\nabla_Y\xi,\ \varphi X)\}=0\,.
\end{aligned}
$$

So, because of our hypotheses, we get from (12) and (13)

$$
(14)\quad \begin{aligned}
&\xi(f)=\xi(h)=0 \qquad h\{g(\nabla_X\xi,\ \varphi X)-g(\nabla_Y\xi,\ \varphi Y)\}=0\\
&h\{3g((\nabla_\xi\varphi)Y,\ X)+2g(\nabla_X\xi,\ \varphi Y)-4g(\nabla_Y\xi,\ \varphi X)\}=0\,.
\end{aligned}
$$

By interchanging to role of X and Y in (14), we obtain

$$
(15)\quad \begin{aligned}
&\xi(f)=\xi(h)=0 \qquad h\{g(\nabla_X\xi,\ \varphi Y)+g(\nabla_Y\xi,\ \varphi X)\}=0\\
&h\{g((\nabla_\xi\varphi)Y,\ X)+2g(\nabla_X\xi,\ \varphi Y)\}=0 \qquad h\{g(\nabla_X\xi,\ \varphi X)-g(\nabla_Y\xi,\ \varphi Y)\}=0\,.
\end{aligned}
$$

Step 3. Let X, Y, W be orthogonal to ξ and $V=Z=\xi$. The same procedure leads to

$$
(16)\quad \begin{aligned}
&Y(f-h)g(X,\ W)-X(f-h)g(Y,\ W)\\
&+h\{g(\nabla_\xi\xi,\ Y)g(X,\ W)-g(\nabla_\xi\xi,\ X)g(Y,\ W)-g((\nabla_\xi\varphi)Y,\ \xi)g(\varphi X,\ W)\\
&+g((\nabla_\xi\varphi)X,\ \xi)g(\varphi Y,\ W)-2g((\nabla_\xi\varphi)\xi,\ W)g(X,\ \varphi Y)\}=0\,.
\end{aligned}
$$

Again put $\|X\|=\|Y\|=1$, Y orthogonal to ξ, X, φX and $W=X$, $W=Y$, $W=\varphi X$, $W=\varphi Y$. Then we get

$$
(17)\quad \begin{aligned}
&X(f-h)+hg(\nabla_\xi\xi,\ X)=0 \qquad Y(f-h)+hg(\nabla_\xi\xi,\ Y)=0\\
&hg((\nabla_\xi\varphi)Y,\ \xi)=0 \qquad hg((\nabla_\xi\varphi)X,\ \xi)=0.
\end{aligned}
$$

Since $g(\varphi X,\ \xi)=0$ this leads to

$$
(18)\quad X(f-h)=0 \qquad hg((\nabla_\xi\varphi)X,\ \xi)=0 \qquad h\nabla_\xi\xi=0\,.
$$

Hence, from (9), (10), (15) and (16) we conclude

(i) *f and h are constant on M;*
(ii) *since* $h \neq 0$

$$g((\nabla_{\varphi Y}\varphi)Y, X) + g((\nabla_Y\varphi)X, \varphi Y) = 0 \qquad g((\nabla_X\varphi)X, \varphi Y) = 0$$

$$(19) \qquad g(\nabla_X\xi, \varphi Y) + g(\nabla_Y\xi, \varphi X) = 0 \qquad g((\nabla_\xi\varphi)Y, X) + 2g(\nabla_X\xi, \varphi Y) = 0$$

$$\nabla_\xi\xi = 0 \qquad g(\nabla_X\xi, \varphi X) - g(\nabla_Y\xi, \varphi Y) = 0.$$

Step 4. Since $g(X, \nabla_X\xi) = 0$, it is easy to derive from (19) that ξ *is a Killing vector field.* Further, the second condition implies

$$(20) \qquad (\nabla_X\varphi)X = \alpha g(X, X)\xi$$

for X orthogonal to ξ and from the last condition in (19) it then follows that α is independent of X.

Now, (4), (5) and (6) imply

$$(21) \qquad R_{X\xi}\xi = -(f-h)X$$

for X orthogonal to ξ. Moreover, since ξ is a Killing vector field, we have (see for example [**1**])

$$(22) \qquad \nabla_X\nabla_Y\xi - \nabla_{\nabla_X Y}\xi = -R_{X\xi}Y$$

and so, with (21)

$$\nabla_{\nabla_X\xi}\xi = -(f-h)X.$$

Hence

$$(23) \qquad g(\nabla_X\xi, \nabla_X\xi) = (f-h)g(X, X)$$

for X orthogonal to ξ. This implies $f-h \geqslant 0$.

Step 5. First, we suppose $f-h=0$. Then (23) yields

$$(24) \qquad \nabla_X\xi = 0$$

which means that ξ *is parallel*. Moreover, from (20) we get

$$(\nabla_X \varphi)X = 0 \tag{25}$$

and from (19) we may conclude that (25) is valid for any vector field X. So

$$(\nabla_X \varphi)Y + (\nabla_Y \varphi)X = 0.$$

Finally proceed as in Step 1 with $Z = X$, $Y = \varphi X$ and W now arbitrary. This leads to

$$(\nabla_X \varphi)Y - (\nabla_Y \varphi)X = 0.$$

(The proof is similar to that of Theorem 12.7 in $[5]_1$). Hence $\nabla\varphi = 0$. From this, we get that *$(M, g, \varphi, \xi, \eta)$ is a co-Kählerian manifold.* Moreover $R = f(\varphi_1 + \varphi_2)$ which implies easily that *$(M, g, \varphi, \xi, \eta)$ is a $C(0)$-space form.*

Step 6. Put $f - h = \alpha^2 > 0$ and

$$\tilde{\varphi}X = -\frac{1}{\alpha}\nabla_X \xi. \tag{26}$$

Then, (22) yields $\tilde{\varphi}^2 X = -X$ for X orthogonal to ξ and hence $\tilde{\varphi}^2 = -I + \eta \otimes \xi$. Moreover,

$$(d\eta)(X, Y) = \frac{1}{2}(g(Y, \nabla_X \xi) - g(X, \nabla_Y \xi)) = -g(X, \nabla_Y \xi) = \alpha g(X, \tilde{\varphi}Y)$$

and so
$$g(\tilde{\varphi}X, \tilde{\varphi}Y) = g(X, Y) - \eta(X)\eta(Y).$$

This all implies that $(g, \tilde{\varphi}, \xi, \eta)$ is an almost contact metric structure on M. Further,

$$(\nabla_X \tilde{\varphi})Y = \nabla_X(\tilde{\varphi}Y) - \tilde{\varphi}\nabla_X Y = -\frac{1}{\alpha}(\nabla_X \nabla_Y \xi - \nabla_{\nabla_X Y}\xi)$$

and so, with (22),
$$(\nabla_X \tilde{\varphi})Y = \frac{1}{\alpha}R_{X\xi}Y.$$

Then, (4) yields

$$(\nabla_X \tilde{\varphi}) Y = \frac{f-h}{\alpha} \{g(X, Y)\xi - \eta(Y)X\} = \alpha\{g(X, Y)\xi - \eta(Y)X\} .$$

This and Proposition 1 imply that $(M, \tilde{\varphi}, g, \xi, \eta)$ is an α-Sasakian manifold.

Further, (19) and (26) imply

$$(\tilde{\varphi}\varphi + \varphi\tilde{\varphi})X = aX$$

for any unit X orthogonal to ξ, where $a = -2g(\varphi X, \tilde{\varphi}X)$ is independent of X. Then (4), (5) and (6) yield

$$R_{X\tilde{\varphi}XX\tilde{\varphi}X} = f + \frac{3}{4} ha^2 = \mu$$

for any unit vector field X orthogonal to ξ. So, $(M, g, \tilde{\varphi}, \xi, \eta)$ is a $C(\alpha^2)$-space form with constant φ-holomorphic sectional curvature μ. This implies

$$R_{XY}Z = \frac{1}{4}(\mu + 3\alpha^2)\tilde{\varphi}_1 + \frac{1}{4}(\mu - \alpha^2)\tilde{\varphi}_2$$

where φ is replaced by $\tilde{\varphi}$ in the expressions (5), (6) for φ_1 and φ_2. Moreover, (4) may be written as

$$R_{XY}Z = \frac{1}{4}(\lambda + 3\alpha^2)\varphi_1 + \frac{1}{4}(\lambda - \alpha^2)\varphi_2 \qquad \text{where } \lambda = f + 3h .$$

Then we get

$$\begin{aligned}(27)\qquad &(\mu - \lambda)\{g(X, Z)Y - g(Y, Z)X + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y\\ &- g(X, Z)\eta(Y)\xi + g(Y, Z)\eta(X)\xi\} + (\mu - \alpha^2)\{g(\tilde{\varphi}Z, Y)\tilde{\varphi}X + g(Z, \tilde{\varphi}X)\tilde{\varphi}Y\\ &- 2g(X, \tilde{\varphi}Y)\tilde{\varphi}Z\} - (\lambda - \alpha^2)\{g(\varphi Z, Y)\varphi X + g(Z, \varphi X)\varphi Y - 2g(X, \varphi Y)\varphi Z\} = 0 .\end{aligned}$$

Now, take $\|X\| = \|Y\| = 1$, X orthogonal to ξ, Y orthogonal to ξ, X, φX and $Z = X$

in (27). We obtain

$$(\mu-\lambda)\,g(Y,\ W)+3(\mu-\alpha^2)\,g(X,\ \tilde{\varphi}Y)\,g(X,\ \tilde{\varphi}W)=0\,. \tag{28}$$

Note that $\mu\neq\alpha^2$ since $\mu=\alpha^2$ implies $\mu=\lambda=\alpha^2$ and hence $h=0$. So, suppose first that $\mu=\lambda$. Then (28) implies $g(\tilde{\varphi}X,\ Y)=0$ and hence $\tilde{\varphi}X=b\varphi X$. This yields at once $\tilde{\varphi}=\pm\varphi$. For $\mu\neq\lambda$ we put $W=\tilde{\varphi}\varphi X$ in (28). We then get

$$g(\varphi X,\ \tilde{\varphi}Y)=0$$

and hence $g(X,\ \tilde{\varphi}Y)=0$. But then, (28) implies by putting $Y=Z$ that $\mu=\lambda$, which is a contradiction.

This concludes the proof of the Theorem.

References

[1] D. BLAIR, *Contact manifolds in Riemannian geometry*, Lectures Notes in Mathematics, **509**, Springer-Verlag, Berlin-Heidelberg-New York, 1976.

[2] P. BUEKEN and L. VANHECKE, *Algebraic characterizations by means of the curvature in contact geometry*, to appear in Proc. III Intern. Symp. Diff. Geom., Peniscola, 1988.

[3] D. JANSSENS and L. VANHECKE, *Almost contact structures and curvature tensors*, Kodai Math. J. 4 (1981), 1-27.

[4] S. MARCHIAFAVA, *Variétés riemanniennes dont le tenseur de courbure est celui d'un espace symétrique de rang un*, C.R. Acad. Sci. Paris Sér. I **295** (1982), 463-466.

[5] F. TRICERRI and L. VANHECKE: $[\bullet]_1$ *Curvature tensors on almost Hermitian manifolds*, Trans. Amer. Math. Soc. **267** (1981), 365-398; $[\bullet]_2$ *Variétés riemanniennes dont le tenseur de courbure est celui d'un espace symétrique irréductible*, C.R. Acad. Sci. Paris **302** (1986), 233-235; $[\bullet]_3$ *Curvature homogeneous Riemannian manifolds*, to appear in Ann. Sci. École Norm. Sup.

[6] K. YANO and M. KON, *Structures on manifolds*, Series in Pure Mathematics, 3, World Scientific Publ. Co., Singapore, 1984.

Abstrait

On caractérise certaines structures de contact à l'aide d'une expression ponctuelle du tenseur de courbure.

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