

MUKUT MANI TRIPATHI (\*)

**Non existence of proper mixed foliated semi-invariant  
submanifolds of Sasakian manifolds (\*\*)**

**1 - Introduction**

Let  $\overline{M}$  be an almost contact metric manifold with structure  $(\phi, \xi, \eta, g)$  (D. E. Blair [2], p. 19-21). A submanifold  $M$  of  $\overline{M}$  is called *semi-invariant* if there exist two differentiable distributions  $D$  and  $D^\perp$  on  $M$  satisfying (A. Bejancu [1], p. 100)

$$TM = D \oplus D^\perp \oplus \{\xi\}$$

where  $D$ ,  $D^\perp$  and  $\{\xi\}$  are mutually orthogonal to each other, the distribution  $D$  is invariant by  $\phi$ , i.e.,  $\phi(D) = D$ , and the distribution  $D^\perp$  is anti-invariant by  $\phi$ , i.e.,  $\phi(D^\perp) \subset T^\perp M$ .

In particular, if  $D$  and  $D^\perp$  are non-trivial then  $M$  is called a *proper semi-invariant* submanifold of  $\overline{M}$ .

In [3], a semi-invariant submanifold is said to be *mixed foliated* if  $D \oplus \{\xi\}$  is integrable and  $h(Z + \xi, X) = 0$  for all  $Z \in D$  and  $X \in D^\perp$ . Moreover, a theorem, regarding the reduction of codimension of proper mixed foliated semi-invariant submanifolds of a Sasakian space form  $\overline{M}(-3)$ , is established.

Here we prove

**Theorem.** *Sasakian manifolds do not admit proper mixed foliated semi-invariant submanifolds.*

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## 2 - Proof of the theorem

Let  $M$  be a semi-invariant submanifold of a Sasakian manifold  $\bar{M}$ . On a Sasakian manifold it follows (D. E. Blair [2], p. 73-74)

$$\bar{\nabla}_X \xi = -\phi X \quad X \in T\bar{M}$$

where  $\bar{\nabla}$  is the Levi-Civita connection of  $g$ . Using Gauss formula we have

$$-\phi X = \nabla_X \xi + h(X, \xi) \quad X \in TM$$

where  $\nabla$  is the induced connexion on  $M$  and  $h$  is the second fundamental form. If  $X \in D^\perp$  then  $\nabla_X \xi = 0$  and

$$h(X, \xi) = -\phi X.$$

Moreover, if  $M$  is mixed foliated then the above equation yields

$$\phi X = 0 \quad X \in D^\perp.$$

Thus  $D^\perp = \{0\}$  and  $M$  can not be a *proper* mixed foliated semi-invariant submanifold.

## References

- [1] A. BEJANCU, *Geometry of CR-submanifolds*, Reidel, Dordrecht, The Netherlands 1986.
- [2] D. E. BLAIR, *Contact manifolds in Riemannian geometry*, Lecture Notes in Math. 509, Springer, Berlin 1976.
- [3] S. M. KHURSHEED HAIDER, V. A. KHAN and S. I. HUSAIN, *Reduction in codimension of mixed foliated semi-invariant submanifold of a Sasakian space form  $\bar{M}(-3)$* , Riv. Mat. Univ. Parma 1 (1992) 147-153.

## Sommario

*Si dimostra che una varietà di Sasaki non ammette sottovarietà semi-invarianti proprie, dotate di fogliazione mista.*

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