Riv. Mat. Univ. Parma (5) 5 (1996), 1-14

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Decomposable expected utility (**)

1 - Introduction

A diffused conviction (see, for instance, M. J. Machina [15], M. Weber and C. Camerer [24]) is that the Expected Utility theory forms a robust system within the range of individual decision making under uncertainty, in spite of the development of alternative models of individual choice under uncertainty *in light of a growing body of evidence that individuals do not maximize expected utility* (M. J. Machina [15]). The descriptive realism, the formal elegance based on the corpus of probability theory and the high *normative appeal* arouse the attention of the decision maker towards the methods of EU theory. Furthermore, among the conceptual motivations that have made EU theory one of the most settled areas in economics, it is worth mentioning the strong connection between EU and the subjectivistic approach in probability: indeed, it is known that utility can be interpreted as a probability (e.g. J. L. Fine [4]; D. V. Lindley [11]).

The independence axiom is simultaneously at the origin of both the rigorous formal setting and the contradictions that sometimes arise between the prescription imposed on the basis of EU and the actual behaviour of the decision maker (although he had all the information about the prescription). The cause of discrepancy is that the preference function is a linear functional on the set of distribution functions. The paradoxes, as the Allais paradox (M. Allais [1]), the common ratio effect (D. Kahneman and A. Tversky [7]) and others (see also the survey paper by M. Weber and C. Camerer [24]), that express empirical failures of the EU theory, synthesize these contradictions and induce some reflections about the weight that the attitude towards the risk has in decision making.

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(**) Received September 22, 1995. AMS classification 90 A 10.

Several generalizations and modifications (e.g. D. Kahneman and A. Tversky [7], M. J. Machina [13], J. Quiggin [18], Schmeidler [19], [20], I. Gilboa [6], M. Yaari [26]; see also Fishburn's book [5] and the paper by E. Karni and D. Schmeidler [8]) of EU theory have been introduced in order to define a formal context to interpret decision making motivations, that can be performed on a wider background, including non-expected behaviour and, for instance, psychological inputs. These non-EU models capture the non-uniform attitude of the single decision maker toward the risk. A right environment, in which to study individual decisions under uncertainty, seems to be a system completing the EU paradigm with the possibility of taking into consideration other components of a single person's behaviour, such as, for instance, unconscious processes.

It is worth recalling the graphical illustration of the property of linearity in probabilities by representing, for all lotteries $(x_1, p_1; x_2, p_2; x_3, p_3)$, $x_1 < x_2 < x_3$, by the points of a triangular diagram, the half unit square having axes p_1 and p_3. In EU scheme the indifference curves are parallel lines. The more risk averse a person is, the steeper is the slope of the indifference curves. As the degree of risk aversion is defined in terms of utility function independent of the probabilities and as the utilities are the same in the whole diagram, then the degree of risk aversion is constant, i.e. independent of the locations of the lotteries (M. J. Machina [14]). The paradoxes mentioned above can be clarified in the triangle diagram by fanning out indifference curves: the lines near the corner $(p_1, p_3) = (1, 0)$ are flatter, while the lines near $(p_1, p_3) = (0, 1)$ are steeper. Several generalized EU models contemplate such a behaviour of the indifference curves in the triangle diagram (for instance D. Kahneman and A. Tversky [7], M. J. Machina [14]).

Our present aim is to define a sufficiently general framework, where the contributions of risk aversion are also taken into account. Such an evaluation is performed in terms of set functions that are decomposable with respect to suitable archimedean operations (decomposable measures), that generalize additive measures; so the context actually includes the probability. Furthermore this approach successfully combines a wide generality with the concrete possibility to perform calculations, as required in the applications.

2 - Aims and background

We have mentioned in Sec. 1 that several different modifications and generalizations of EU theory have been proposed. D. Schmeidler ([19], [20]) introduced a non-additive model that uses the integration operation due to Choquet

[2], that is an actual generalization of Lebesgue's one. Analogous tools have been investigated and applied by I. Gilboa [6] and P. Wakker [23].

Another integration theory with respect to suitable decomposable set functions, that gives rise to a different actual extension of Lebesgue's theory too, is due to S. Weber [25]. We shall deal with the attitudes toward risk and some paradoxical phenomena (Sec. 4), and the concept of certainty equivalent (Sec. 5) in this general framework. Let us recall some definitions and results (S. Weber [25], B. Schweizer and A. Sklar [21]) in order to introduce Weber's integral.

A binary operation $\perp$ on the real unit interval $J = [0, 1]$ is said to be a *t-conorm* if $\perp$ is non decreasing in each argument, associative, commutative, and has 0 as neutral element. A t-conorm is said to be *archimedean*, if it is continuous and such that $\perp(x, x) > x$ for every x in the open interval $(0, 1)$. An archimedean t-conorm is called *strict*, if it is strictly increasing in the open square $(0, 1)^2$. The following representation theorem holds

Theorem 1. (C. H. Ling [12]). *A binary operation $\perp$ on J is an archimedean t-conorm, if and only if there exists a strictly increasing and continuous function $g\colon J \to [0, \infty]$, with $g(0) = 0$, such that*

$$\perp(x, y) = g^{(-1)}(g(x) + g(y))$$

vhere $g^{(-1)}$ denotes the pseudo-inverse of g defined by

$$g^{(-1)}(x) = g^{-1}(\min(x, g(1))).$$

Moreover $\perp$ is strict, if and only if $g(1) = \infty$.

The function g, called an *additive generator* of $\perp$, is unique up to a positive constant factor. The following identity holds

$$g(g^{(-1)}(x)) = \min(x, g(1)). \tag{1}$$

Example (M. Sugeno [22], S. Weber [25]). For $w > -1$,

$$U_w(a, b) = \min(a + b + wab, 1) \tag{2}$$

with a, b in J, defines a non-strict t-conorm U_w with additive generator

$$g_w(x) = \frac{1}{w}\ln(1 + wx) \tag{3}$$

whose pseudo-inverse is given by

(4) $$g_w^{-1}(y) = \begin{cases} \frac{1}{w}((\exp(wy) - 1) \\ 1 \end{cases}$$

according to whether $y \leqslant g_w(1)$, or $y \geqslant g_w(1)$. In particular, we have $U_0(a, b) = \min(a + b, 1)$, with $g_0(x) = x$.

Let (Ω, β) be a measurable space. A set function $m\colon \beta \to J$, with $m(\emptyset) = 0$ and $m(\Omega) = 1$, is called (S. Weber [25]) a decomposable measure with respect to a t-conorm $\perp$, or a $\perp$*-decomposable measure*, if $m(A \cup^0 B) = m(A) \perp m(B)$, where $\cup^0$ denotes disjoint union. Moreover m is called a σ-$\perp$*-decomposable measure*, if $m(\cup_{n \geqslant 1}^0 A_n) = \perp_{n \geqslant 1} m(A_n)$, where the union above is disjoint and countable. The notation $(\sigma$-$)\perp$-decomposable will stand for $\perp$ or σ-$\perp$-decomposable.

The classification (S. Weber [25]) of $(\sigma$-$)\perp$-decomposable measures with respect to archimedean t-conorms depends on the property of the operation $\perp$ to be or not strict. In fact, if $\perp$ is *strict*, then we have

S $g \circ m\colon \beta \to [0, +\infty]$ is an infinite (σ-) additive measure, whenever m is a $(\sigma$-$)\perp$-decomposable one.

If $\perp$ is *non-strict archimedean*, then one of the following cases occur:

NSA $g \circ m\colon \beta \to [0, g(1)]$ is a finite (σ-) additive measure such that $(g \circ m)(\Omega) = g(1)$

NSP $g \circ m$ a finite measure with $(g \circ m)(\Omega) = g(1)$, which is only pseudo-($\sigma$-) additive, i.e. it is possible that

$$(g \circ m)(\cup_{n \geqslant 1}^0 A_n) = g(1) < \sum (g \circ m)(A_n).$$

Let now $\perp$ be an Archimedean t-conorm with additive generator g and m a σ-$\perp$-decomposable measure on the measurable space (Ω, β). Let $f\colon \Omega \to J$ be any measurable function with the normality condition $0 \leqslant f(x) \leqslant 1$.

Definition (S. Weber [25]). The *integral of f over A* is defined for the two cases:

A. Except for **NSP** with $m(A) = 1$, let

$$\int_A f \perp m = g^{-1}\left(\int_A f \, \mathrm{d}(g \circ m)\right)$$

P. For **NSP** with $m(A) = 1$, Ω is assumed to be m-achievable (i.e. there exists a sequence $\{A_n\}$, with $m(A_n) < 1$, such that $\Omega = \cup_{n \geqslant 1}^0 A_n$), then let

$$\int_A f \perp m = g^{(-1)}\left(\sum \int_{A \cap A_n} f \, \mathrm{d}(g \circ m)\right).$$

In case **A** the induced measure $g \circ m$ is additive and the integral is a Lebesgue integral. In the second part **P** stands for *pseudoadditivity* of $g \circ m$ and the definition is independent of the special choice of $\{A_n\}$.

Properties of the functional $I: f \to \int_\Omega f \perp m$ are

i. $I(1_A) = m(A)$, in particular $I(0) = 0$ and $I(1) = 1$

ii. $h \geqslant f$ implies $I(h) \geqslant I(f)$

iii. $1 \geqslant f + h$ implies $I(f + h) = I(f) \perp I(h)$

iv. $f_n \uparrow f$ a.e. implies $I(f_n) \uparrow I(f)$

v. if $f_n \to f$ a.e. with the furher assumption in case S that there exists a function h such that $h \geqslant f_n$ and $I(h) < 1$, or in case **NSP**, $\sum(g \circ m)(A_n) < \infty$, then $I(f_n) \to I(f)$.

Relations between Choquet's and Weber's integrals have been stated (S. Weber [25]). The Choquet's integral is additive for comonotonic functions (C. Dellacherie [3], D. Schmeidler [19], [20]), while in general this is not the case, so it could be suggested to use Weber's integral because of the persistence of several formal properties of Lebesgue's integral and decomposability property **iii**, that allows to combine the single expectations.

3 - Expectation

3.1 - The formal setting

Let us introduce a non-EU model making use of Weber's integral. We consider a set of lotteries with outcomes between a and b, $-\infty < a < b < +\infty$. We can therefore suppose from now on, without loss of generality, that $vN - M$ utility function u fulfills normality condition $0 \leqslant u(x) \leqslant 1$ on $[a, b]$. If the value of the functional

$$E_\perp(u) = I(u) = \int_A f \perp m \tag{5}$$

is assumed as the numerical utility index representing preferences, then the complete ordering axiom for preferences over lotteries is preserved. Furthermore a continuity requirement (M. Weber and C. Camerer [24]) is satisfied: indeed, due to the continuity of g, g^{-1} and u, given the lotteries X, Y, Z, such that X is preferred or indifferent to Y and Y is preferred or indifferent to Z, there exists a probability p such that Y is indifferent to $pX + (1-p)Z$. In general independence does not hold, because of the loss of linearity in probabilities.

The following equivalence, that holds for integrals I with respect to decomposable measures

$$\int u \perp m_1 \geqslant \int u \perp m_2 \text{ if and only if } m_1(y \leqslant x) \leqslant m_2(y \leqslant x)$$

for any increasing utility function u, $0 \leqslant u(x) \leqslant 1$, replaces the fulfilment of stochastic dominance principle. It is worth recalling (see Sec. 2) that in case **NSA** the composition $g \circ p$ is a probability measure. This takes account of the subject's alteration of p by means of the function g and restores a new probability measure $g \circ p$. For sake of simplicity let us deal, in the examples, with discrete probability distributions $P = (p_i)$ over a fixed outcome set $\{x_i\}$. Therefore, the preference functional (5) assumes the form $E_\perp(u) = I(u) = g^{-1}(\sum u_i g(p_i))$ in the case **A** of the definition. In case **P**, $E_\perp(u) = I(u) = g^{(-1)}(\sum_n \sum_{A_n \ni x_i} u(x_i)\, g(p_i))$, where the A_n's belong to the power set of Ω.

Thus the behaviour of the decision maker consists in choosing a suitable additive generator g and maximizing the preference functional, i.e., solving the optimization problem $\max(E_\perp(u))$, with the maximum taken over the lotteries. Furthermore equation (5), in case **A**, yields a standard expectation with respect to the additive measure $g \circ m$ (up to the order preserving inversion g^{-1}).

Let us consider an example related to the case that the set function $g \circ m$ is finite and pseudo-additive. Let $\perp(a, b) = \min(a + b, 1)$, $\Omega = N$ be the set of positive integers, β the power set of N, and the measure of a subset A of N be defined by $m(A) = \min(\frac{a}{n^*}, 1)$, with a the number of elements of A and fixed n^* in N. Then m is σ-$\perp$-decomposable with additive generator $g(x) = x$ and $g^{(-1)}(y) = \min(y, 1)$. One has $g \circ m = m$, that is pseudo-σ-additive. Indeed, given the disjoint subsets A and B of N, with b the number of elements of B, and $a + b > n^*$, it results $g \circ m(A \cup B) = 1$, and

$$g \circ m(A) + g \circ m(B) = \min(\frac{a}{n^*}, 1) + \min(\frac{b}{n^*}, 1) > 1 = g(1).$$

An application of such an evaluation is as follows. It is known that, when counting red corpuscles in 1 mm^3 of human blood, what is important is that the threshold of 5 millions be approximately reached. Now, for instance, if c red corpuscles have been counted in a given sample C, with $c > 5{,}000{,}000 = n^*$, the value of the measure associated to the sample is $g \circ m(C) = \min(\frac{c}{n^*}, 1) = 1$. Let $u(x)$ denote the utility to have x red corpuscles (in 1 mm^3 of blood), say an increasing function over N, taking constant value 1 in a suitable right neighbourhood of 5,000,000; therefore for the expectation one has $u(c)\, g \circ m(C) = 1 = u(n^*)$.

Let us recall that alterations in the probabilities p were also suggested (J. Quiggin [18] and M. Yaari [26]) by means of modifications in the probability distributions, that reflect the adjustment in the perception of the decision maker. In the prospect theory by D. Kahneman and A. Tversky [7] a preference function which is separable into the components is defined. The preference function here defined in terms of Weber's integral, what enables us to deal with nonlinear preferences and infinite or continuous outcome set, is expressed in terms of Lebesgue's integral with respect to a composition $g \circ m$ (that is a (σ-) additive measure).

In the case that $g \circ m$ is a measure on a segment $A = [a, b]$, generated by an absolutely continuous monotone function F, one gets

$$\int_A u \, \mathrm{d}(g \circ m) = \int_A u(x) \, \mathrm{d}F(x) = \int_A u(x) F'(x) \, \mathrm{d}x .$$

Remark that an upper bound for $E_\perp$ when g is convex is given by

$$E_\perp(u) \leqslant \int g^{-1} \circ u \, \mathrm{d}(g \circ m) = E(g^{-1} \circ u)$$

where E denotes (additive) expectation with respect to the measure $g \circ m$.

3.2 - Indifference curves

Let us suppose now u strictly monotone and twice differentiable, with second derivative continuous. If r_1 and r_2 denote the risk aversion degrees of g^{-1} and u^{-1}, respectively, and

$$r_1 > r_2 , \tag{6}$$

then $g^{-1} \circ u$ is a risk averse utility function (J. W. Pratt [17], R. L. Keeney and H. Raiffa [9]). Thus, if g and u satisfy (6) near the certaintly, a behaviour that is coherent with the maximization of $E_\perp$ takes risk aversion into account. This fact suggests to investigate whether the maximum principle, in the present non-additive setting, can help to describe some paradoxical patterns.

To this purpose let us consider the t-conorm U_w. The expected utility of the lottery $L = (u_1, p_1; u_2, p_2; u_3, p_3)$, using the probability $g_w \circ p$ instead of p, is given by

$$E_\perp(L) = g_w^{-1}(\textstyle\sum u_i g_w(p_i)) = (\frac{1}{w})(\exp(\sum u_i \ln(1 + wp_i)) - 1) \tag{7}$$

or $E_\perp(L) = 1$, following (4).

Equation (7) entails that individual indifference curves in the (p_1, p_3) triangular diagram are represented by

$$(1 + wp_1)^{u_1}(1 + wp_2)^{u_2}(1 + wp_3)^{u_3} - 1 = cw \tag{8}$$

with constant c and $p_2 = 1 - p_1 - p_3$.

In order to get some information about the shape of the indifference curves (8), let us consider the special case $0 \neq h = (1 + wp_2)^{u_2}$. By putting $k = (wc + 1)h^{-1}$, $H = k^{1/u_3}$, and $s = \dfrac{u_1}{u_3}$, equation (8), for $w \neq 0$, assumes the form

$$p_3 = Hw^{-1}(1 + wp_1)^{-s} - w^{-1}. \tag{9}$$

Some computational difficulties remain even in the simplified equation (9): let just observe that H is a function of w. However we can easily find that, for $-1 < w < 0$, the function (9) $p_3 = p_3(p_1)$ is decreasing and concave, $p_1 < -\dfrac{1}{w}$ and the whole diagram (when the variables range into the reals) has the vertical asymptote $p_1 = -\dfrac{1}{w}$ (> 1) and the horizontal asymptote $p_3 = -\dfrac{1}{w}$.

For instance, when $w = -0.5$, $c = 0.6$, $u_1 = 0.8$, it is $p_1 = \sim 0.54$ and the intersection of the whole diagram with p_3 axis is ~ 1.83.

The curve can be plotted like in Figure 1.

For $w > 0$, the function p_3 in (9) is decreasing and convex, $p_1 > -w^{-1}$ (< 0) and vertical and horizontal asymptotes have equations $p_1 = -w^{-1}$ and $p_3 = -w^{-1}$.

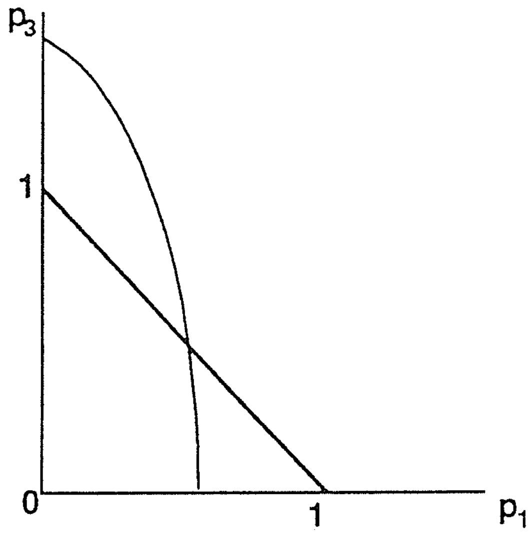

Fig. 1.

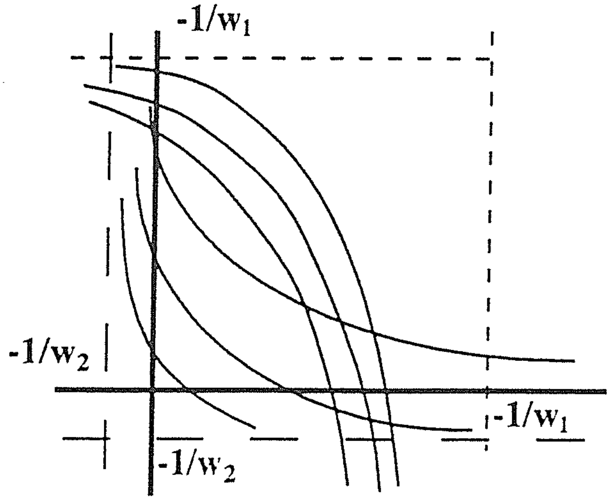

Fig. 2.

In Figure 2 some indifference curves for negative w and positive w are plotted. Fanning out is represented by the two families of curves: precisely the tendency for risk aversion is mainly associated with the values $w < 0$, while that for risk seeking with the values $w > 0$. These behaviours are related with the action of the inverse function of g_w that is concave or convex according to negative or positive w, respectively.

Some comments are now in order. The present non-EU model becomes more fllexible because of the twofold transformation: that one originated by the utility function and the other due to the transformation of the probability. This allows to take into account different psychological aspects of the decision maker. The utility transformation acts over the wealth W of the individual and his risk attitude: evaluating a lottery X means to compute z in equation $E(u(W + X)) = u(W + z)$.

The EU model captures how the subject evaluates the consequences of the presence of X over his economic situation. The t-conorm operator transforms probability evaluations, so that optimistic or pessimistic behaviours of the decision maker can be incorporated. More precisely, on a first level two mental attitudes of the subject must be put about the consequences and the relations between the decision maker and the destiny. The consequences of a lottery influence the utility function by means of the transformation of the monetary amounts; therefore we get a risk love or a risk aversion that is economic in nature. The decision maker, when elaborating the synthesis procedure, transforms probability evaluations, obtaining a risk love or a risk aversion that is psychological in nature.

On a second level we look at the effects of the two components, for instance observing how the decision maker estimates lotteries. If the combined effects of the two transformations lead to $z < E(X)$, then the subject reveals risk aversion *tout court*. Such a risk aversion may come either from a double aversion or from the occurrence of a single aversion, provided that this one prevails over the other.

4 - Common ratio effect and Allais paradox

Let now deal with some decision problems, that put in evidence failures of the classical EU paradigm. We shall rank the preferences between lotteries by the decision rule derived from the maximand E_{U_w}. It is possible to explain cases included in the general pattern termed the *common ratio effect* and the *Allais paradox*.

Les us start first by recalling that the common ratio effect is a phenomenon involving pairs of prospects of the form:

$$L = (0, 1-p; X, p; Y, 0) \quad \text{versus} \quad M = (0, 1-q; X, 0; Y, q)$$

$$L' = (0, 1-rp; X, rp; Y, 0) \quad \text{versus} \quad M' = (0, 1-rq; X, 0; Y, rq)$$

for $0 < r < 1$, $p > q$ and $0 < X < Y$. The classical EU model prescribes choices of L and L', if the indifference lines are steep, or else M and M', if they are flat, whereas a systematic tendency of the individuals for choices to deviate from these prescriptions is observed. For instance (D. Kahneman and A. Tversky [7]), assuming that $X = 0.3$, $Y = 0.4$, $p = 1$, $q = 0.8$, $r = 0.25$ (X and Y are expressed in units of 10,000$), the greatest part of polled individuals prefer L over M and M' over L'. By applying (8) to this numerical case, we get, for $w = -0.5$

$$E_{\perp}(L) = g^{-1}(-2(0.3)\ln 0.5) = \sim g^{-1}(0.416)$$

$$E_{\perp}(M) = g^{-1}(-2(0.4)\ln 0.6) = \sim g^{-1}(0.409),$$

and from monotonicity of g^{-1}, the preference of L over M; while, for $w = 0.5$

$$E_{\perp}(L') = g^{-1}(0.6\ln 1.125) = \sim g^{-1}(0.071)$$

$$E_{\perp}(M') = g^{-1}(0.8\ln(1.1) = \sim g^{-1}(0.076)$$

i.e. a preference of M' over L'.

It is worth remarking that a preference of M' over L' is induced also by the value $w = -0.5$, indeed $E_{\perp}(L') = \sim g^{-1}(0.080)$, $E_{\perp}(M') = \sim g^{-1}(0.084)$.

An analogous setting of the preferences is performed when dealing with the Allais paradox:

$$a = (0, 0; 0.1, 1; 0.5, 0) \quad \text{versus} \quad b = (0, 0.01; 0.1, 0.89; 0.5, 0.1)$$

and $$c = (0, 0.9; 0.1, 0; 0.5, 0.1) \quad \text{versus} \quad d = (0, 0.89; 0.1, 0.11; 0.5, 0).$$

Contrary to the preference settings under EU hypothesis, we get from (8), for $w = -0.5$

$$E_{\perp}(a) = \sim g^{-1}(0.138) \qquad E_{\perp}(b) = \sim g^{-1}(0.118)$$

$$E_{\perp}(c) = \sim g^{-1}(0.051) \qquad E_{\perp}(d) = \sim g^{-1}(0.011).$$

This exhibits a preference of a over b and c over d, which agrees with the modal preferences of the subjects.

Of course the tediousness of the computation of a non-expected utility increases when linearity is abandoned. For instance, in order to evaluate also risk attitudes, we have to face the problem of indifference curve tracing. This can be done only by a detailed analysis of equation (9) and a systematic plotting of indifference curves. Different t-conorms can be used and, in any case, the resulting model has to be investigated, whether it can *fit the data better than the standard* EU *model* (M. J. Machina [15]). To this purpose the user, e.g., the decision analyst or the decision maker, of such a non-EU model should be in a position to tune the different parameters at his disposal and select the indifference curves. That can be easily done by a simple inspection of the drawings, which can be automatically plotted.

5 - Certainty equivalent

Let X be a random money amount having support X and $vN - M$ like utility function u. Following the setting due to I. H. LaValle [**10**] and L. Peccati [**16**], we shall deal with the concepts of *certainty equivalent* from both the seller's and the buyer's sides, in our non-additive context.

The concepts of Seller Certainty Equivalent (**SCE**) and Buyer Certainty Equivalent (**BCE**) will be now defined, following the classical pattern, by the equations in z

$$u(z) = E_{\perp}(u(X)) \qquad u(0) = E_{\perp}(u(X - z))$$

respectively.

Let m denote a σ-$\perp$-decomposable measure on (Ω, β) and u a utility function, whose range is the interval $J = [0, 1]$. We are now able to prove the following existence theorems for **SCE** and **BCE**. We shall also state uniqueness conditions in particular cases.

Theorem 2. *If the utility function u fulfills the conditions:*

i. *u is defined in an interval $[a, b]$ that includes X*

ii. *u is continuous and strictly monotone*

then the **SCE** *for X exists; in cases* S *and* **NSA** *it is unique*

Proof. The result follows from conditions **i** and **ii** and strict monotonicity and continuity of g.

Theorem 3. *Let X be bounded and $x_0 = \inf(X)$ and $x_1 = \sup(X)$. If the utility function u fulfills the following conditions:*

i. *u is defined at least on the interval $[a, b]$, where $a = x_0 - x_1$ and $b = x_1 - x_0$*

ii. *u is continuous and strictly monotone*

then the **BCE** *exists. It is unique in cases* S *and* **NSA**.

Proof. Let us put $w(z) = E_{\perp}(u(X - z))$. By properties v and **ii** (Sec. 2), w is continuous over $[a, b]$ and monotone. The theorem follows from continuity of u.

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Sommario

Si propone un «nonexpected utility model» per la rappresentazione delle preferenze, nel quale si usa un'operazione di integrazione non additiva, ma decomponibile rispetto a una conveniente operazione archimedea (una conorma triangolare). Si mostra che alcune deviazioni dal modello dell'utilità attesa, come il paradosso di Allais e l'effetto del rapporto comune, si possono descrivere entro questo paradigma.

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