Introduction

SIMAI, the Italian Society of Applied and Industrial Mathematics, connects applied math researchers and industry professionals to develop mathematical models for real industrial problems. Its focus is to promote applied mathematics research that can be directly transferred from academia to industry.

In the last decade, SIMAI members have recognized the importance of schools in shaping a practical vision of mathematics. This vision emphasizes mathematics as strongly connected to real-world issues and providing concrete solutions. This idea does not stand alone; several national standards documents, as well as the PISA framework, recognize that mathematical modelling is a key competence with great relevance for modern societies.

Due to these considerations, researchers in applied mathematics and mathematics education have initiated EduSIMAI, a satellite one-day event integrated into the biennial SIMAI conference. The objective is to facilitate collaboration among school teachers, mathematics education researchers, mathematicians, and industry professionals. The focus is on reflecting upon and jointly designing teaching trajectories specifically tailored to mathematical modelling, which involves applying mathematical principles to solve real-world problems. As Rellensmann and Schukajlow argue in this issue, "it is the aim of mathematics education to equip students with the abilities to use their mathematical knowledge in a flexible manner so that they can develop solutions to unsolved problems and create new ideas and innovations" (p. 1). Gaio (this issue) further emphasizes that introducing students to mathematical concepts beyond the standard curriculum enhances motivation and imparts a sense of reality to mathematics.

Mathematical modeling requires creativity, which is understood as "a student's abilities to generate novel, unusual, and appropriate ideas and outcomes during problem solving (Rellensmann & Schukajlow, this issue, p. 2).However, Vasquez, Barquero, and Bosch (this issue) note that "besides all the progress made in research and the support of educational policies and curriculum reforms, well-established modeling activities do not disseminate as expected" (p. 2), and Geisler (this issue) further notes that this challenge extends to model validation, as "some students do not see validation as a necessary step in modeling processes" (Geisler, this issue, p. 1). As a matter of fact, thus, the goal of transforming the teaching and learning of mathematics in our schools is far from being reached. School mathematics still relies on traditional, formal, and abstract approaches to mathematics, approaches that make the subject detached from the students' real world. In particular, Vasquez, Barquero, and

Bosch argue that there is a well-established trend consisting of organizing "the teaching and learning of school disciplines on the logic of (monodisciplinary) concepts rather than the logic of (multidisciplinary) problems" (p. 2). Pezzutto (this issue) provides the example of a teacher struggling to introduce a teaching method that is based on gamification and modeling, providing further evidence of the obstacles that even motivated teachers face every day in this respect.

This Special Issue of the Rivista di Matematica dell'Università di Parma aims to address the challenge of improving the teaching of mathematical modeling in schools. Each paper offers a thorough literature review of existing research on mathematical modeling and, then, expands important aspects that need to be addressed in order to enhance the way modeling is taught in school.

Rellensmann and Schukajlow emphasize the role of creativity in mathematical modelling. Their study, involving grade 10 students and utilizing an eye-tracker, specifically explores the construction and use of self-generated drawings to address modelling problems. They investigate the development of originality, fluency, and a sense of usefulness throughout the modelling process. Findings indicate that students generated highly original ideas during various stages of drawing. Detailed drawings stimulated reflections on additional parameters, enhancing originality. However, inaccuracies in drawing led to inadequate mathematical models and solutions (i.e., a low level of usefulness).

Gaio explores how a didactical unit on cryptography can promote modelling abilities. This study delves into the effectiveness of a hands-on activity in cryptography in terms of improving students' engagement and self-efficacy. Additionally, it assesses the perceived usefulness of mathematics and evaluates the efficiency of inquiry-based teaching in fostering conceptual understanding.

Geisler focuses on the validation phase of modelling, that is the process where students reflect on the meaning and correctness of the obtained mathematical results within real-world problem contexts. According to existing literature, this is far from easy for students. Geisler suggests that the most effective way to develop validation competence is through tasks combining mathematical modelling with experiments. In this kind of task, students perform experiments to collect data that can be used for subsequent modelling. Examining a large group of grade 10 students, Geisler investigates how students validate their models using experimental data. The study reveals that students employ diverse criteria for model validation, showing that sensible validation is not self-evident in the context of mathematical modelling with experiments.

Vasquez, Barquero, and Bosch show the results of an interdisciplinary experience involving grade 10 students learning mathematics, biology, and verbal expression on the topic of COVID-19. The students were involved in developing researchable questions and in attempting to answer them. The findings reveal that the topic and the questions posed by the students were clearly of social relevance, and the utility of the final answers was beyond doubt. At the same time, the interdisciplinarity of the questions and their treatment were also visible, proving this approach to teaching to be effective in promoting a prospective that integrates different disciplines to address a real-world situation.

Diaz, Romo-Vazquez, and Sanchez-Aguilar conduct their research in the context of engineering education, specifically focusing on the mathematical modelling of hydraulic systems in civil engineering. The study explores activities in engineers' workplaces that can be adapted to engineering courses through appropriate mathematical modelling teaching proposals. The researchers also highlight the conditions necessary for integrating these proposals into engineering courses. Results indicate that while engineering students can perform the activities, their limited experience hampers proposed solutions, involving costly materials, non-existent pipe diameters, and a lack of practical verification.

Bassi and Brunetto finally address the issue of how mathematical modeling competencies can be assessed for both students and teachers. Working with a small sample of grade 11 students, their qualitative study allows the researchers to conclude that, when a real-world problem is addressed by students working in groups, a scientific report is an effective evaluation instrument for assessment. The limitations concerning the joint coding procedure and the sample size need to be overcome, but the interesting distinction between competence and competency and the identification of the competencies specific for the seventh step in the modelling cycle are the central results of this contribution.

Reflecting on the crucial aspects of learning mathematical modeling cannot be detached from reflections on how to increase teachers' knowledge, confidence and beliefs about the use of modeling activities in their mathematics classes. Two contributions in this issue focus on this delicate topic.

Barelli, Barquero, and Branchetti share findings from the European research project IDENTITIES. This project focuses on interdisciplinarity in secondary pre-service teacher education, developing materials and a unified approach that integrates various theoretical perspectives in mathematics, physics, and computer science education. In the paper, the researchers present the design principles of an interdisciplinary Study and Research Path for Teacher Education about the modelling of the evolution of COVID-19, aimed at fostering interdisciplinary reflection among prospective teachers with different backgrounds.

Andrà and Martignone explore the possible features that modeling activities can have when they are carried out in primary school and they focus on the knowledge teachers can resort to in these early grades. Beside the important role of teacher's specialised mathematical knowledge, the study also highlights the strong interconnection between beliefs and teaching choices, as well as how a particular modeling activity holds the potential of modifying a teacher's beliefs about teaching-learning mathematics as well as consolidating her pedagogical content knowledge.

Pezzutto explores the tensions that teachers face when they introduce modeling activities in their mathematical classes, and analyzes the forces that can prevent teachers leaving traditional, abstract, rote approaches to mathematics and its teaching at school and at university. Students' requests and reactions remain at the center of a teacher's concerns, more that her beliefs about what "works" in a mathematics lesson.

This special issue attempts to present a diverse scenario of both teaching and learning mathematical modelling. Its goal is to inform teachers and researchers about the potentials and limitations of various approaches to this topic across different school levels and contexts. It does not represent, in the Editors' view, and ending point, but rather a starting one – an open window on the issues that are cogent for the teaching of applied mathematics, and that need a special attention.

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Chiara Andrà and Laura Branchetti